Progression and Series: Question

CONCEPTUAL QUESTIONS

Single Answer Type Questions

1. The p th term of an A.P. is a and q th term is b. then the sum of its' (p+ q) terms is

(A) $\frac{(p+q)}{2}\left[a+b+\frac{a-b}{p-q}\right]$ (B) $\frac{(p-q)}{2}\left[a+b-\frac{a-b}{p-q}\right]$

(C) $\frac{p+q}{2}\left[a-b+\frac{p-q}{a-b}\right]$ (D) $\frac{p-q}{2}\left[a+b+\frac{p-q}{a+b}\right]$

2. If the sum of m terms of an arithmetical progression is equal to the sum of either the next n terms or the next p terms, then $\left(\frac{n+m}{n-m}\right)\left(\frac{p-m}{p+m}\right)$ is

(A) $\frac{n}{p}$ (B) $\frac{p}{n}$ (C) np (D) $\frac{p}{m}$

3: If S_n denotes the sum to n terms of a G.P. whose first term and common ratio are a and r(r $\neq$ 1) respectively, then $S_1 + S_2 + S_3 + \ldots + S_n$ is

(A) $\frac{na}{1-r}$ (B) $\frac{na}{1-r}-\frac{ar(1-r^n)}{(1-r)^2}$

(C) $\frac{a}{1-rn}$ (D) $\frac{a}{1-r^n}-\frac{r(1-r^n)}{(1-r)^2}$

4. If |x| < 1 and |y| < 1 then $(x + y) + (x^2 + xy + y^2) + (x^3 + x^2y + xy^2 + y^3) + \ldots\ldots \infty$ is

(A) $\frac{x+y-xy}{(1-x)(1-y)}$ (B) $\frac{x-y-xy}{(1-x)(1-y)}$

(C) $\frac{x+y-xy}{(1+x)(1+y)}$ (D) $\frac{x-y-xy}{(1+x)(1+y)}$

5. If a, b and c be in G.P. and x, y be the arithmetic means between a, b and b, c respectively then $\frac{a}{x}+\frac{c}{y}$ is

(A) 2 (B) 1
(C) 3 (D) 4

6. In a centre test, there are p questions, in this 2^{p-r} students give wrong answers to at least r questions $(1 \le r \le p)$. If total number of wrong answers given is 2047, then the value of p is

(A) 14 (B) 13 (C) 12 (D) 11

7. Let a, b, c be three distinct positive real numbers in G.P., then $a^2 + 2bc - 3ac$ is
(A) >0 (B) <0 (C) =0 (D) cann't be found out

8 If a, b, x, y are positive natural numbers such that $\frac{1}{x}+\frac{1}{y}=1$ then $\frac{a^x}{x}+\frac{b^y}{y}$ is
(A) $\leq ab$ (B) $\geq ab$ (C) =ab (D) cann't be found out

9. The maximum value of $a^2 b^3 c^4$ subject to a + b + c = 18 is
(A) $4^2 6^3 8^4$ (B) $4^3 6^2 8^4$
(C) $4^4 6^2 8^3$ (D) $4^2 6^4 8^3$

10. If the 1st and ther $(2n-1)^{th}$ term of an A.P., G.P., and H.P. are equal and their nth terms are a,b and c respectively , then
(A) $a=b=c$ (B) $a \leq b \leq c$
(C) $a+c=b$ (D) $2ac-b^2=0$

SINGLE CORRECT CHOICE QUESTIONS

LEVEL-I

A.P., G.P., H.P. and Mean

1. If the sum of first 2n terms of A.P. 2, 5, 8,... is equal to the sum of the first n terms of the A.P. 57, 59, 61, ..., then n equals

(A)10 (B) 12
(C) 11 (D) 13

2. If S denotes the sum to infinity and S_n the sum of n terms of the series $1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+.....$,

such that $S - S_n < \frac{1}{1000}$, then the least value of n is

(A) 8 (B) 9
(C) 10 (D) 11

3. If a_1, a_2, a_3, ... is an A.P. such that

$a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24} = 225$,

then $a_1 + a_2 + a_3 + ... + a_{23} + a_{24}$ is equal to

(A) 909 (B) 75
(C) 750 (D) 900

4. If the sum of first p terms, first q terms and first r terms of an A.P. be a, b and c respectively, then $\frac{a}{p}(q-r)+\frac{b}{q}(r-p)+\frac{c}{r}(p-q)$ is equal to

(A) 0 (B) 2
(C) pqr (D) $\frac{8abc}{pqr}$

5. The numbers $3^{2\sin 2\theta - 1}$, 14, $3^{4-2\sin 2\theta}$ form first three terms of an A.P. Its fifth term is equal to

(A) –25 (B) –12
(C) 40 (D) 53

6. If sum of n terms of an A.P. is $3n^2 + 5n$ and $T_m = 164$, m = ?

(A) 26 (B) 27
(C) 28 (D) 25

7. If the sum of the series $2+\frac{5}{x}+\frac{25}{x^2}+\frac{125}{x^3}+.........$ is finite, then

(A) $|x| > 5$ (B) $-5 < x < 5$
(C) $|x| < 5/2$ (D) $|x| > 5/2$

8. If a, b, c, d are in H.P,, then ab + bc + cd is equal to

(A) 3 ad (B) (a + b) (c + d)

(C) ac (D) none of these

9. The sum of integers from 1 to 100 which are divisible by 2 or 5 is

(A) 300 (B) 3050

(C) 3200 (D) 3250

10. The interior angles of a polygon are in arithmetic progression. The smallest angle is 120^0 and the common difference is 5. The number of sides of the polygon is

(A) 7 (B) 9

(C) 11 (D) 16

11. If a_1, a_2, a_3 ….. a_n are in H.P., then $\frac{a_1}{a_2 + a_3 + ...a_n}, \frac{a_2}{a_1 + a_3 + ... + a_n}, ... \frac{a_n}{a_1 + a_2 + ... + a_{n-1}}$ are in

(A) A.P. (B) G.P.

(C) H.P. (D) none of these

12. If x > 0 and $\log_2 x + \log_2\left(\sqrt{x}\right) + \log_2\left(\sqrt[4]{x}\right) + \log_2\left(\sqrt[8]{x}\right) + \log_2\left(\sqrt[16]{x}\right) + ... = 4$ then x equals

(A) 2 (B) 3

(C) 4 (D) 5

13. If the ratio of sum of m terms and n terms of an A.P. be $m^2 : n^2$, then the ratio of its m^{th} and n^{th} terms will be

(A) 2m –1 : 2n – 1 (B) m : n

(C) 2m + 1 : 2n + 1 (D) none

14. The ratio between the sum of n terms of two A.P.'s is 3n + 8 : 7 n + 15. Then the ratio between their 12^{th} terms is

(A) 5 : 7 (B) 7 : 16

(C) 12 : 11 (D) none

15. $\log_3 2$, $\log_6 2$, $\log_{12} 2$ are in

(A) A.P. (B) G.P.

(C) H.P. (D) none

16. In a G.P. if the $(m + n)^{th}$ term be p and $(m – n)^{th}$ term be q, then its m^{th} term is

(A) $\sqrt{(pq)}$ (B) $\sqrt{(p/q)}$

(C) $\sqrt{(q/p)}$ (D) p/q

17. Between 1 and 31 m arithmetic means are inserted so that the ratio of the 7^{th} and $(m–1)^{th}$ means is 5 : 9. Then the value of m is

(A) 12 (B) 13

(C) 14 (D) 15

A.G.P., V_n Medhod

18. If $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}-\frac{1}{11}+.......=\frac{\pi}{4}$, then value of $\frac{1}{1.3}+\frac{1}{5.7}+\frac{1}{9.11}+..........$ is

(A) $\frac{\pi}{8}$ (B) $\frac{\pi}{6}$

(C) $\frac{\pi}{4}$ (D) $\frac{\pi}{36}$

19. Sum to n terms of the series $\frac{1}{(1+x)(1+2x)}+\frac{1}{(1+2x)(1+3x)}+\frac{1}{(1+3x)(1+4x)}+..........$ is

(A) $\frac{nx}{(1+x)(1+nx)}$ (B) $\frac{n}{(1+x)\left[1+(n+1)x\right]}$

(C) $\frac{x}{(1+x)\left(1+(n-1)x\right)}$ (D) $\frac{nx}{(1+x)[1+(n+1)x]}$

20. Sum of the series $S=1+\frac{1}{2}(1+2)+\frac{1}{3}(1+2+3)+\frac{1}{4}(1+2+3+4)+.........$ upto 20 terms is

(A) 110 (B) 111

(C) 115 (D) 116

21. If $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+......\text{upto } \infty=\frac{\pi^2}{6}$, then value of $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+..........$ upto to ∞ is

(A) $\frac{\pi^2}{4}$ (B) $\frac{\pi^2}{6}$

(C) $\frac{\pi^2}{8}$ (D) $\frac{\pi^2}{12}$

22. Sum of the series $S = 1^2 - 2^2 + 3^2 - 4^2 + ... - 2002^2 + 2003^2$ is

(A) 2007006 (B) 1005004

(C) 2000506 (D) 1005040

Inequalities

23. If three positive real numbers a, b, c are in A.P., with abc = 4, then the minimum value of b is

(A) $4^{1/3}$ (B) 3

(C) 2 (D) 1/2

24. If x, y and z are positive real numbers such that x + y + z = a then

(A) $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\geq\frac{9}{a}$

(B) $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}<\frac{9}{a}$

(C) $(a-x)(a-y)(a-z)>\frac{8}{27}a^3$

(D) $(a-x)(a-y)(a-z)>a^3$

25. If a, b and c are distinct positive real numbers and $a^2 + b^2 + c^2 = 1$, then $ab + bc + ca$ is

(A) less than 1

(B) equal to 1

(C) greater than 1

(D) any real number

LEVEL -II

Inequalities

26. The greatest value of $x^2y^3z^4$, (if $x + y + z = 1$, $x, y, z > 0$) is

(A) $\frac{2^9}{3^5}$

(B) $\frac{2^{10}}{3^{15}}$

(C) $\frac{2^{15}}{3^{10}}$

(D) $\frac{2^{10}}{3^{10}}$

27. If a, b and c are three positive real numbers, then the minimum value of the expression

$\frac{b+c}{a}+\frac{c+a}{b}+\frac{a+b}{c}$ is

(A) 1

(B) 2

(C) 3

(D) 6

A.P., G.P., H.P. & Mean

28. If S be the sum, P the product and R the sum of the reciprocals of n terms of a G.P., then

$\left(\frac{S}{R}\right)^n =$

(A) P

(B) P^2

(C) P^3

(D) $\sqrt{P}$

29. If x, y, z be respectively the pth, qth and rth terms of G.P., then

$(q-r)\log x + (r-p)\log y + (p-q)\log z =$

(A) 0

(B) 1

(C) –1

(D) 2

30. In a G.P., $T_2 + T_5 = 216$ and $T_4 : T_6 = 1:4$ and all terms are integers, then its first term is

(A) 16

(B) 14

(C) 12

(D) 15

31. If the roots of the equation $x^3 - 12x^2 + 39x - 28 = 0$ are in A.P., then their common difference will be

(A) ±1 (B) ±2

(C) ±3 (D) ±4

32. If a, b, c, d are nonzero real numbers such that

$(a^2 + b^2 + c^2)(b^2 + c^2 + d^2) \le (ab + bc + cd)^2$, then a, b, c, d are in

(A) AP (B) GP

(C) HP (D) AGP

33. Let $a_1, a_2, a_3, \ldots$ be in AP and a_p, a_q, a_r be in GP. Then $a_q : a_p$ is equal to

(A) $\frac{r-p}{q-p}$ (B) $\frac{q-p}{r-q}$

(C) $\frac{r-q}{q-p}$ (D) 1

34. Three distinct real numbers a, b, c are in G.P. such that a + b + c = x b, then

(A) $0 < x < 1$ (B) $-1 < x < 3$

(C) $x < -1$ or $x > 3$ (D) $-1 < x < 2$

35. If x, y, z are in G.P., $a^x = b^y = c^z$, then

(A) $\log_c b = \log_a c$ (B) $\log_1 c = \log_b c$

(C) $\log_a b = \log_c b$ (D) $\log_b a = \log_c b$

36. The rth, sth and t^{th} terms of a certain G.P. are R, S and T respectively, then the value of $R^{s-t} S^{t-r} . T^{r-s}$ is

(A) 0 (B) 1

(C) –1 (D) 2

37. Three numbers form an increasing G.P. If the middle number is doubled, then the new numbers are in A.P. The common ratio of G.P. is

(A) $2-\sqrt{3}$ (B) $2+\sqrt{3}$

(C) $\sqrt{3}-2$ (D) 2

38. If one geometric mean G and two arithemtic means p and q be inserted between two numbers, then G^2 is equal to:

(A) $(3p - q)(3q - p)$ (B) $(2p - q)(2q - p)$

(C) $(4p - q)(4q - p)$ (D) $(4p + q)(4q + p)$

39. If the first and (2n+ 1)th terms of an A.P.; G.P. and H.P. are equal and their(n+1)th terms are a, b and c respectively, then

(A) $a > b > c$ (B) $ac = b^2$

(C) $a + b = c$ (D) $a + c = b$

40. The sum of the two numbers is $2\frac{1}{6}$. An even numbers of arithmetic means are inserted between them and their sum exceeds their number by 1. Then the number of means inserted is

(A) 6 (B) 8
(C) 12 (D) 15

A.G.P., V_n Method

41. The sum upto (2n + 1) terms of the series $a^2 - (a + d)^2 + (a + 2d)^2 - (a + 3d)^2 + ...$ is

(A) $a^2 + 3nd^2$ (B) $a^2 + 2nad + n(n - 1)d^2$
(C) $a^2 + 3nad + n(n - 1)d^2$ (D) $a^2 + 2nad + n(2n + 1)d^2$

42. $\sum_{n=1}^{\infty} \frac{1}{(n+1)(n+2)(n+3)....(n+k)}$ is equal to

(A) $\frac{1}{(k-1)\lfloor k-1}$ (B) $\frac{1}{k\lfloor k}$
(C) $\frac{1}{(k-1)\lfloor k}$ (D) $\frac{1}{\lfloor k}$

43. The sum of first n terms of the series $1^2 + 2 \times 2^2 + 3^2 + 2 \times 4^2 + 5^2 + 2 \times 6^2 + ...$ is $n(n + 1)^2/2$ when n is even. When n is odd the sum of the series is

(A) $n^2(3n + 1)/4$ (B) $n^2\frac{(n+1)}{2}$
(C) $n^3(n - 1)/2$ (D) none of these

44. If $\sum_{r=1}^{n} t_r = \frac{1}{12}n(n+1)(n+2)$, then value of $\sum_{r=1}^{n}\frac{1}{t_r}$ is

(A) $\frac{2n}{n+1}$ (B) $\frac{n-1}{(n+1)!}$
(C) $\frac{4n}{(n+1)}$ (D) $\frac{3n}{n+2}$

45. Sum to n terms of the series $\frac{1}{5!}+\frac{1!}{6!}+\frac{2!}{7!}+\frac{3!}{8!}+..........$ is

(A) $\frac{2}{5!}-\frac{1}{(n+1)!}$ (B) $\frac{1}{4}\left(\frac{1}{4!}-\frac{n!}{(n+4)!}\right)$
(C) $\frac{1}{4}\left(\frac{1}{3!}-\frac{3!}{(n+2)!}\right)$ (D) $\frac{1}{4}\left(\frac{1}{4!}+\frac{n!}{(n+4)!}\right)$

46. Sum to n terms of the series $\frac{1}{1.2.3}+\frac{3}{2.3.4}+\frac{5}{3.4.5}+\frac{7}{4.5.6}+....$ is

(A) $\frac{n(n+1)}{2(n+2)(n+3)}$ (B) $\frac{n(3n+1)}{4(n+1)(n+2)}$

(C) $\frac{1}{6}-\frac{5}{(n+1)(n+4)}$ (D) $\frac{(3n+1)}{(n+1)(n+2)}$

47. Sum to n terms of the series $\frac{1^3}{1}+\frac{1^3+2^3}{1+3}+\frac{1^3+2^3+3^3}{1+3+5}+...........$is

(A) $\frac{n}{24}(n^2+9n+14)$ (B) $\frac{n}{24}(2n^2+7n+15)$

(C) $\frac{n}{24}(2n^2+9n+13)$ (D) $\frac{n}{24}(n^2+11n+12)$

Inequalities

48. Let $p,q,r \in R^+$ and $27pqr \ge (p+q+r)^3$ and $3p+4q+5r=12$ then $p^3 + q^4 + r^5$ is equal to

(A) 3 (B) 6

(C) 2 (D) 1

49. If a, a_1, a_2, a_3, ... a_{2n-1}, b are in AP,a, b_1 , b_2 ,b_3 ..., b_{2n-1}, b are in GP and a, c_1, c_2, c_3, ... ,c_{2n-1}, b are in HP, where a, b are positive, then the equation $a_nx^2 - b_nx + c_n = 0$ has its roots

(A) real and unequal (B) real and equal

(C) imaginary (D) none of these

50. In an acute angled triangle ABC, the minimum value of $\tan^n A + \tan^n B + \tan^n C$. is (when $n \in N, n > 1$)

(A) $3^{\frac{n}{2}}$ (B) 3^n

(C) $3^{\frac{n}{2}+1}$ (D) $3^{\frac{n}{2}-1}$

LEVEL -III

51. If $A, A_1, A_2, ..., A_{2n}, B$ be an A. P. ; A, $G_1, G_2, ..., G_{2n}, B$ be a G.P. and H is the harmonic mean of A and B, then $\frac{A_1+A_{2n}}{G_1G_{2n}}+\frac{A_2+A_{2n-1}}{G_2G_{2n-1}}+....+\frac{A_n+A_{n+1}}{G_nG_{n+1}}$ is equal to

(A) $\frac{2n}{H}$ (B) 2nH

(C) nH (D) $\frac{n}{H}$

52. Given that $0 < x < \frac{\pi}{4}$ and $\frac{\pi}{4} < y < \frac{\pi}{2}$ and $\sum_{k=0}^{\infty}(-1)^k \tan^{2k} x = p;\ \sum_{k=0}^{\infty}(-1)^k \cot^{2k} y = q;$ then $\sum_{k=0}^{\infty}\tan^{2k} x \cot^{2k} y$ is

(A) $\frac{1}{p}+\frac{1}{q}-\frac{1}{pq}$ (B) $\frac{1}{\left\{\frac{1}{p}+\frac{1}{q}-\frac{1}{pq}\right\}}$

(C) p + q – pq (D) p + q + pq

53. The sum of the series $\frac{8}{5}+\frac{16}{65}+.........+\frac{128}{2^{18}+1}$ is

(A) $\frac{540}{1088}$ (B) $\frac{1088}{545}$

(C) $\frac{1001}{500}$ (D) $\frac{1013}{545}$

54. If $f(r)=1+\frac{1}{2}+\frac{1}{3}+.........+\frac{1}{r}$ and $f(0)=0$, then value of $\sum_{r=1}^{n}(2r+1)f(r)$

(A) $n^2 f(n)$ (B) $(n + 1)^2 f(n+1) - \frac{n^2+3n+2}{2}$

(C) $(n +1)^2 f(n) - \frac{n^2+n+1}{2}$ (D) $(n +1)^2 f(n)$

55. Solution of the system of equation $2x^4 = y^4 + z^4$, $xyz = 8$
knowing that the logarithms $\log_y x$, $\log_z y$, $\log_x z$ form a geometric progression is

(A) x = y = z = 2 (B) x = y = 2 , z = 3

(C) x = 2, y = z = 3 (D) x = y = z= 3

56. If a, b, c be distinct positive are in G.P. and $\log_c a$, $\log_b c$, $\log_a b$ be in A.P., then common difference of this A.P. is .

(A) 3/2 (B) 1/2

(C) 2 (D) 5/2

57. The nth term of a series is given by $t_n=\frac{n^5+n^3}{n^4+n^2+1}$ and if sum of its n terms can be expressed as $S_n=a_n^2+a+\frac{1}{b_n^2+b}$, where a_n and b_n are the nth terms of some arithmetic progressions and a, b are some constants, then $\frac{b_n}{a_n}$ equal to.

(A) $n\sqrt{2}$ (B) $\frac{n}{\sqrt{2}}$

(C) $\frac{1}{2}$ (D) 2

58. If $a_1, a_2, a_3, ... a_n$ are in A.P., where $a_i > 0$ for all i , then

$$\frac{1}{\sqrt{a_1}+\sqrt{a_2}}+\frac{1}{\sqrt{a_2}+\sqrt{a_3}}+...+\frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}}$$ is equal to

(A) $\frac{1}{\sqrt{a_1}+\sqrt{a_n}}$ (B) $\frac{1}{\sqrt{a_1}-\sqrt{a_n}}$

(C) $\frac{n}{\sqrt{a_1}-\sqrt{a_n}}$ (D) $\frac{n-1}{\sqrt{a_1}+\sqrt{a_n}}$

59. 111 .... 1 (91 times) is a

(A) Prime number (B) Composite number

(C) Not a integer (D) integer

60. A three digit number whose consecutive numbers form a G.P. If we subtract 792 from this number, we get a number consisting of the same digits written in the reverse order. Now if we increase the second digit of the required number by 2, the resulting number will form an A.P. Then the number is

(A) 139 (B) 927

(C)931 (D) 763

MORE THAN ONE CORRECT CHOICE QUESTIONS

LEVEL - I

1. The sum of the first n term (n > 1) of an A.P. is 155 and the common difference is 2. If the first term is a positive integer, then

(A) n can not be even (B) n can not be odd

(C) 5 (D) 6

2. The sum of the numerical series $\frac{1}{\sqrt{3}+\sqrt{7}}+\frac{1}{\sqrt{7}+\sqrt{11}}+\frac{1}{\sqrt{11}+\sqrt{15}}+\text{.........upto n terms}$, is

(A) $\frac{\sqrt{3+4n}-\sqrt{3}}{4}$ (B) $\frac{4}{\sqrt{3+4n}+\sqrt{3}}$

(C) less than n (D) greater than $\sqrt{n}/2$

3. If b_1, b_2, b_3 ($b_1 > 0$) are three successive terms of a G.P. with common ratio r, the value of r for which the inequality $b_3 > 4b_2 - 3b_1$ holds is given by

(A) r > 3 (B) r < 1

(C) r = 3.5 (D) r = 5.2

4. All the term of an A. P. are natural numbers and the sum of the first 20 terms is greater than 1072 and less than 1162. If the sixth term is 32 then

(A) first term is 12 (B) first term is 7

(C) common difference is 4 (D) common difference is 5

5. In a GP the product of the first four terms is 4 and the second term is the reciprocal of the fourth term. The sum of the GP up to infinite terms is

(A) 8 (B) -8

(C) 8/3 (D) –8/3

6. The numbers $\frac{\sin x}{6}$, cos x and tan x will be in G.P. if

(A) $x=\frac{\pi}{3}$ (B) $x=\frac{5\pi}{6}$

(C) $x=\pm\frac{\pi}{3}+2K\pi$ (D) $\pm\frac{\pi}{6}+2K\pi$

7. The next term of the geometric progression x, $x^2 + 2$, $x^3 + 10$ is

(A) 0 (B) 54

(C) $\frac{729}{16}$ (D) $\frac{16}{729}$

8. If sum of n terms of an A.P. is given by $S_n = a + bn + cn^2$ where a, b, c are independent of n, then

(A) $a = 0$

(B) common difference of A.P. must be 2b

(C) common difference of A.P. must be 2c   (D) All above

9. If a, b, c are in A.P., then $2^{ax+1}, 2^{bx+1}, 2^{cx+1}$, $x \neq 0$ are in

(A) A.P.   (B) G.P. when $x > 0$

(C) G.P. if $x < 0$ (D) G.P. for all $x \neq 0$

10. If $a_1, a_2, ... a_n$ are in H.P. and d be the common difference of the corresponding A.P. then the expression $a_1a_2 + a_2a_3 + ... + a_{n-1}a_n$ is equal to

(A) $\frac{a_1 - a_n}{d}$ (B) $(n - 1)(a_1 - a_n)$

(C) $n(a_1 - a_n)$ (D) $(n - 1)a_1a_n$

11. Let the harmonic mean and the geometric mean of two positive numbers be in the ratio 4 : 5. Then the two numbers are in the ratio

(A) 1 : 4   (B) 4 : 1

(C) 3 : 4   (D) 4 : 3

12. Between two unequal numbers, if a_1, a_2 are two AMs; g_1, g_2 are two GMs and h_1, h_2 are two HMs then $g_1.g_2$ is equal to

(A) a_1h_1 (B) a_1h_2

(C) a_2h_2 (D) a_2h_1

13. Let the sum of the series $\frac{1}{1^3} + \frac{1+2}{1^3+2^3} + ... + \frac{1+2+.........+n}{1^3+2^3+..........+n^3}$ upto n terms be S_n, n = 1, 2, 3, ... Then S_n cannot be greater than

(A) 1/2   (B) 1

(C) 2   (D) 4

14. If p, q, r are positive and are in A.P. the roots of the quadratic equation $px^2 + qx + r = 0$ are all real for

(A) $\left|\frac{r}{p} - 7\right| \geq 4\sqrt{3}$ (B) $\left|\frac{q}{p} - 4\right| \geq 2\sqrt{3}$

(C) $\left|\frac{p}{q} - 4\right| \geq 4\sqrt{3}$ (D) $\left|\frac{p}{q} - 1\right| \geq 4\frac{\sqrt{3}}{2}$

15. Let a and b be two positive real numbers. Suppose A_1, A_2 are two arithmetic means; G_1, G_2 are two geometric means and H_1, H_2 are two harmonic means between a and b then

(A) $\frac{G_1G_2}{H_1H_2}=\frac{A_1+A_2}{H_1+H_2}$ (B) $\frac{G_1G_2}{H_1H_2}-\frac{5}{9}=\frac{2}{9}\left(\frac{a}{b}+\frac{b}{a}\right)$

(C) $\frac{H_1+H_2}{A_1+A_2}=\frac{9ab}{(2a+b)(a+2b)}$ (D) $\frac{G_1G_2}{H_1H_2}=\frac{H_1+H_2}{A_1+A_2}$

16. The value of 100 $\left[\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+......+\frac{1}{99.100}\right]$

(A) is an integer (B) lies between 50 and 98

(C) is 100 (D) 99

17. Let S_n = (1) (5) + (2) (5^2) + (3) (5^3) + ... + (n) (5^n) $=\frac{1}{16}\left[(4n-1)5^a+b\right]$, then

(A) a = n + 1 (B) a = n

(C) b = 5 (D) b = 25

18. Four numbers are such that the first three are in A.P., while the last three in G.P.. If the first number is 6 and common ratio of G.P. is $\frac{1}{2}$ then the

(A) sum of first and last number is 7 (B) numbers are 6, 8, 4, 2

(C) numbers are 6, 10, 14, 4 (D) numbers are 6, 4, 2, 1

19. Let S_1, S_2, ... be squares such that for each $n \geq 1$, the length of a side of S_n equals to the length of a diagonal of S_{n+1}. If the length of a side of S_1 is 10 cm, then for which of the following value (s) of n is the area of S_n less than 1 sq. cm?

(A) 7 (B) 8

(C) 9 (D) 10

20. Let x_1, x_2, ... be positive integers in A.P., such that $x_1 + x_2 + x_3 = 12$ and $x_4 + x_6 = 14$. Then x_5 is

(A) a prime number (B) 11

(C) 13 (D) 7

<u>LEVEL - II</u>

21. If a, b, c are in H.P. , then the expression $E=\left(\frac{1}{b}+\frac{1}{c}-\frac{1}{c}\right)\left(\frac{1}{c}+\frac{1}{a}-\frac{1}{b}\right)$ equals

(A) $\frac{2}{bc}-\frac{1}{b^2}$ (B) $\frac{1}{4}\left(\frac{3}{c^2}+\frac{2}{ca}-\frac{1}{a^2}\right)$

(C) $\frac{3}{b^2}-\frac{2}{ab}$ (D) none of these

22. If positive numbers a, b, c, d are in harmonic progression and $a \neq b$, then

(A) a + d > b + c is always true (B) a + b > c + d is always true

(C) a + c > b + d always true (D) ad > bc

23. The sum of the first n terms of the series

$1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + ...$is $\frac{n(n+1)^2}{2}$, when n is even. When n is odd, the sum is

(A) $\frac{n^2(n+1)}{2}$ (B) $\frac{(n+1)^2 n^2}{2}$

(C) even, if odd 'n' is of the type 4 l + 1. (D) even, if the odd 'n' is of the type 4l + 3

24. The three sides of a right-angled triangle are in G.P.. The tangents of the two acute angles may be

(A) $\frac{\sqrt{5}+1}{2}$ and $\frac{\sqrt{5}-1}{2}$ (B) $\sqrt{\frac{(\sqrt{5}-1)}{2}}$

(C) $\sqrt{5}$ and $\frac{1}{\sqrt{5}}$ (D) $\sqrt{\frac{(\sqrt{5}+1)}{2}}$

25. If a, b, c are in A.P., and a^2, b^2, c^2 are in H.P., then

(A) a = b = c (B) $a^2 = b^2 = \frac{c^2}{2}$

(C) a, b, c are in G.P. (D) $\frac{-a}{2}, b, c$ are in G.P.

26. If $(m + 1)^{th}$, $(n + 1)^{th}$ and $(r + 1)^{th}$ terms of an A.P. are in G.P. and m, n, r are in H.P., then the ratio of the first term of the A.P. to its common difference is

(A) $-\frac{n}{2}$ (B) $-\frac{m}{2}$

(C) r (D) $-\frac{mr}{m+r}$

27. If roots of $x^3 + bx^2 + cx + d = 0$ are

(A) in A.P. then $2b^3 - 9bc + 27d = 0$ (B) in G.P. then $b^3d = c^3$

(C) in G.P. then $27d^3 = 9bcd^2 - 4c^3d$ (D) equal then $c^3 = b^3 + 3bc$

28. x_1, x_2, x_3, ... is an infinite sequence of positive integers in G.P. such that $x_1x_2x_3x_4 = 64$. Then the value of x_5 is

(A) is a perfect square (B) is not a perfect square

(C) 128 (D) 16

29. Let a sequence $\{a_n\}$ be defined by $a_n = \frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{n+3} + ... + \frac{1}{3n}$; Then

(A) $a_2 = \frac{7}{12}$

(B) $a_2 = \frac{19}{20}$

(C) $a_{n+1} - a_n = \frac{9n+5}{(3n+1)(3n+2)(3n+3)}$

(D) $a_{n+1} - a_n = \frac{-2}{3(n+1)}$

30. If $\sum_{r=1}^{n} r(r+1)(2r+3)$ = $an^4 + bn^3 + cn^2 + dn + e$, then

(A) a + c = b + d

(B) e = 0

(C) a, b – 2/3, c –1 are in A.P.

(D) c/a is an integers

LEVEL - III

31. Let a_1, a_2, a_3, ... be terms of an A.P.

If $\frac{a_1 + a_2 + ... + a_p}{a_1 + a_2 + ... + a_q} = \frac{p^2}{q^2}, p \neq q$ then $\frac{a_6}{a_{21}}$ must be

(A) less than 1

(B) $\frac{2}{7}$

(C) $\frac{11}{41}$

(D) $\frac{7}{2}$

32. If the numbers $\frac{a+b}{1-ab}$, b and $\frac{b+c}{1-bc}$ are in A.P., then

(A) a, b, c are in HP

(B) a, b^{-1}, c are in HP

(C) a^{-1}, b, c^{-1} are in AP

(D) a, b, c are in GP

33. If a, b, c are in H.P. then

(A) $\frac{a}{b+c-a}, \frac{b}{c+a-b}, \frac{c}{a+b-c}$ are in H.P.

(B) $\frac{2}{b} = \frac{1}{b-a} + \frac{1}{b-c}$

(C) $a - \frac{b}{2}, \frac{b}{2}, c - \frac{b}{2}$ are in G.P.

(D) $\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ are in H.P.

34. The numbers 1, 5, 25 can be three terms (not necessarily consecutive) of

(A) at least one A.P.

(B) at least one G.P.

(C) infinite number of A.P's

(D) infinite number of G.P.'s

35. Let $S_n = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + ... + \frac{1}{n^2}$ and $T_n = 2 - \frac{1}{n,}$ then

(A) $S_2 < T_2$

(B) If $S_k < T_k$ then $S_{K+1} < T_{k+1}$

(C) $S_n < T_n$ for all $n \geq 2$

(D) $S_n > T_n$ for all $n \geq 2007$

LINKED COMPREHENSION TYPE

This section contains 4 paragraphs. Based upon each paragraph, 3 multiple choice questions have to be answered. Each question has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

I. If $x_1, x_2, \ldots\ldots x_n$ are 'n' positive real numbers; then A.M. ≥ G.M. ≥ H.M.

$$\frac{x_1 + x_2 + \ldots\ldots x_n}{n} \geq (x_1 x_2 \ldots\ldots x_n)^{1/n} \geq \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \ldots\ldots + \frac{1}{x_n}}$$ equality occurs when numbers are same using this concept.

1. If a > 0, b > 0, c > 0 and the minimum value of $a(b^2 + c^2) + b(c^2 + a^2) + c(a^2 + b^2)$ is λabc, then λ is

(A) 1 (B) 2

(C) 3 (D) 6

2. If a, b, c, d, e, f are positive real numbers such that a + b + c + d + e + f = 3, then

x = (a + f)(b + e)(c + d) satisfies the relation

(A) $0 < x \leq 1$ (B) $1 \leq x \leq 2$

(C) $2 \leq x \leq 3$ (D) $3 \leq x \leq 4$

3. If a and b are two positive real numbers, and a + b = 1, then the greatest value of a^3b^4 is

(A) $\frac{3^2 4^3}{75}$ (B) $\frac{3^3 4^4}{7^7}$

(C) $\frac{7^7}{3^3 4^4}$ (D) none of these

II. The sum of the squares of three distinct real numbers, which are in G.P., is S^2. If their sum is αS then answers the following questions.

4. α^2 lies

(A) (1/3, 2) (B) (1, 2)

(C) (1/3, 3) (D) none of these

5. If $\alpha^2 = 2$, then value of r equals

(A) $\frac{1}{2}\left(5 - \sqrt{3}\right)$ (B) $\frac{1}{2}\left(3 + \sqrt{5}\right)$

(C) $\frac{1}{2}\left(\sqrt{5} + \sqrt{3}\right)$ (D) $\frac{1}{3}\left(\sqrt{3} + \sqrt{5}\right)$

6. If we drop the condition that the G.P. is strictly increasing and take $\alpha^2 = 3$, then common ratio is given by

(A) $\pm\sqrt{2}$ (B) +1

(C) 0 (D) $\pm\sqrt{3}$

III. Suppose x_1, x_2 be the roots of $ax^2 + bx + c = 0$ and x_3, x_4 be the roots of $px^2 + qx + r = 0$.

7. If $x_1, x_2, \frac{1}{x_3}, \frac{1}{x_4}$ are in A.P. , then $\frac{b^2 - 4ac}{q^2 - 4pr}$ equals

(A) a^2/r^2 (B) b^2/q^2

(C) c^2/p^2 (D) a^2/p^2

8. If a, b, c are in G.P. as well as x_1, x_2, x_3, x_4 are in G.P. then p, q, r are in

(A) A.P. (B) G.P.

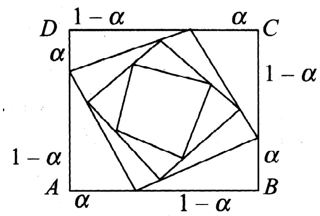
(C) H.P. (D) A.G.P.

9. If x_1, x_2, x_3, x_4 are in G.P., then its common ratio is

(A) $\left(\frac{ar}{cp}\right)^{1/4}$ (B) $\left(\frac{cr}{ap}\right)^{1/3}$

(C) $\sqrt{\frac{cr}{ap}}$ (D) $\sqrt{\frac{ap}{bq}}$

IV. Let ABCD is a unit square and $0 < \alpha < 1$. Each side of the square is divided in the ratio $\alpha : 1 - \alpha$, as shown in figure. These points are connected to obtain another square. The sides of new square are divided in the ratio $\alpha : 1 - \alpha$ and points are joined to obtain another square. The process is continued indefinitely. Let a_n denote the length of side and A_n the area of the nth square



10. The value of α for which $\sum_{n=1}^{\infty} A_n = \frac{8}{3}$ is

(A) 1/3, 2/3 (B) 1/4, 3/4

(C) 1/5, 4/5 (D) 1/2

11. The value of α for which side of nth square equals the diagonals of (n + 1)th square is

(A) 1/3 (B) 1/4

(C) 1/2 (D) $1/\sqrt{2}$

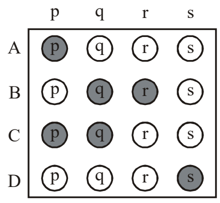
12. If $\alpha = 1/4$ and P_n denotes the perimeter of the nth square then $\sum_{n=1}^{\infty} P_n$ equals

(A) 8/3 (B) 32/3

(C) 16/3 (D) none of these

MATRIX–MATCH TYPE

Statements (A, B, C, D) in **Column I** have to be matched with statements (p, q, r, s) in **Column II**.
The answers to these questions have to be appropriately bubbles as illustrated in the following example.
If the correct matches are A–p, A–s, B–q, B–r, C–p, C–q and D–s, then the correctly bubbled 4 × 4 matrix should be as follows :



1. Match the value of x on the left with the value on the right.

Column I	**Column II**
(A) $5^2\, 5^4\, 5^6 \,.......5^{2x} = (0.04)^{-28}$	(p) $3 \log_3 5$
(B) $x^2 = (0.2)^{\log_{\sqrt{5}}\left(\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+..........\right)}$	(q) 4
(C) $x = (0.16)^{\log_{5/2}\left(\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+..........\right)}$	(r) 2
(D) $3^{x-1} + 3^{x-2} + 3^{x-3} + ...$	(s) 7
$= 2\left(5^2 + 5 + 1 + \frac{1}{5} + \frac{1}{5^2} + ...\right)$	(t) even integer

2. Match the conditions for the equation $ax^3 + bx^2 + cx + d = 0$ having roots in

Column I	**Column II**
(A) AP	(P) $b^3d = ac^3$
(B) GP	(Q) $27ad^3 = abcd^2 - 2c^3\, d$
(C) HP	(R) $2b^3 - 9abc + 27a^2d = 0$
(D) $3\alpha = \beta + \gamma$	(S) $4ad - bc = 0$
	(T) $ad + bc$

3. Let a, b, c, p > 1 and q > 0. Suppose a, b, c are in G.P.

Column I	**Column II**
(A) $\log_p a, \log_p b, \log_p c$ are in	(p) G.P.
(B) $\log_a p, \log_b p, \log_c p$ are in	(q) A.G.P.
(C) $a \log_p c, b \log_p b, c \log_p a$	(r) H.P.
(D) $q^{\log_a p}, q^{\log_b p}, q^{\log_c p}$ are in	(s) A.P.

4. Let α, β, γ be three numbers such that $\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}=\frac{1}{2}, \frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2}=\frac{9}{4}$,

and $\alpha + \beta + \gamma = 2$.

Column I	**Column II**
(A) $\alpha\beta\gamma$	(p) 6
(B) $\beta\gamma + \gamma\alpha + \alpha\beta$	(q) 8
(C) $\alpha^2 + \beta^2 + \gamma^2$	(r) –2
(D) $\alpha^3 + \beta^3 + \gamma^3$	(s) –1
	(t) even integer

ASSERTION – REASON TYPE

Each question contains STATEMENT – 1 (Assertion) and STATEMENT – 2 (Reason). Each question has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

Instructions:

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True; Statement–2 NOT a correct explanation for Statement-1.

(C) Statement–1 is True, Statement–2 is False

(D) Statement –1 is False, statement–2 is True.

1. For $r \geq 1, \text{ and } x \neq 1 \text{ Let } t_r = 1 + 2x + 3x^2 + ... + rx^{r-1}$

STATEMENT-1 : Sum of $t_1 + t_2 + ... + t_n$ is $\frac{n\left(1+x^{n+1}\right)}{(1-x)^2} - \frac{2x\left(1-x^n\right)}{(1-x)^3}$

STATEMENT-2 : For $r \geq 1, \text{ and } x \neq 1, 1 + x + x^2 + ... + x^{r-1} = \frac{1-x^r}{1-x}$ and

$1 + 2x + 3x^2 + ... + rx^{r-1} = \frac{1-x^r}{(1-x)^2} - \frac{rx^r}{1-x}$

2. Let a, b, c be three positive real numbers which are in H.P.

STATEMENT-1 : $\frac{a+b}{2a-b} + \frac{c+b}{2c-b} \geq 4$

STATEMENT-2 : If x > 0, then $x + \frac{1}{x} \geq 4$

3. **STATEMENT-1 :** If a, b, c are three positive real numbers such that $a + c \neq b$ and

$\frac{1}{a} + \frac{1}{a-b} + \frac{1}{c} + \frac{1}{c-b} = 0$ then a, b, c are in H.P.

because

STATEMENT-2 : If a, b, c are distinct positive real numbers such that

a (b – c)x^2 + b (c – a) xy + c (a – b)y^2 is a perfect square, then a, b, c are in H.P.

4. **STATEMENT-1 :** There exists no A.P. whose three terms are $\sqrt{3}, \sqrt{5} \text{ and } \sqrt{7}$

because

STATEMENT-2 : If t_p, t_q and t_r are three distinct terms of an A.P., then $\frac{t_r - t_p}{t_q - t_p}$ is a rational number

5. Let a, b, c be three distinct non-zero real numbers

STATEMENT-1 : If a, b, c are in A.P. and b, c, a are in G.P., then c, a, b are in H.P.

STATEMENT-2 : If a, b, c are in A.P. and b, c, a are in G.P. than a : b : c = 4 : 2 : –1

6. **STATEMENT-1 :** If three positive numbers in G.P. represent sides of a triangle, then the common ratio of the G.P. must lie between $\frac{\sqrt{5}-1}{2}$ and $\frac{\sqrt{5}+1}{2}$

STATEMENT-2 : Three positive real number can form sides of a triangle if sum of any two is greater than the third

7. Let a, b, c, d are four positive number

Statement–1 : $\left(\frac{a}{b}+\frac{b}{c}\right)\left(\frac{c}{d}+\frac{d}{e}\right) \ge 4\sqrt{\frac{a}{e}}$

Statement–2 : $\frac{b}{a}+\frac{c}{b}+\frac{d}{c}+\frac{e}{d}+\frac{a}{e} \ge 5$.

8. Let a, b, c and d be distinct positive real numbers in H.P.

Statement–1 : a + d > b + c

Statement–2 : $\frac{1}{a}+\frac{1}{d}=\frac{1}{b}+\frac{1}{c}$

9. **STATEMENT-1 :** $\frac{1^2}{(1)(3)}+\frac{2^2}{(3)(5)}+............+\frac{n^2}{(2n-1)(2n+1)}=\frac{n(n+1)}{2(2n+1)}$

because

STATEMENT-2 : $\frac{1}{(1)(3)}+\frac{1}{(3)(5)}+.........+\frac{1}{(2n-1)(2n+1)}=\frac{1}{2n+1}$

10. **STATEMENT-1 :** There exists no A.P. whose three terms are $\sqrt{3}, \sqrt{5}$ and $\sqrt{7}$

because

STATEMENT-2 : If t_p, t_q and t_r are three distinct terms of an A.P., then $\frac{t_r - t_p}{t_q - t_p}$ is a rational number

INTEGER ANSWER TYPE QUESTIONS

1. An infinite G.P. is selected from $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}$ to converge to 1/7. If $1/2^a$ is the first term of such a G.P., find a.

2. Find the smallest natural number m > 90 for which $n = \underbrace{111......1}_{m\,times}$ is not a prime number. Hence find the value of m-87.

3. Suppose a, x, y, z and b are in A.P. when x + y + z = 15, and a, α, β, γ, b are in H.P.. when $1/\alpha + 1/\beta + 1/\gamma = 5/3$. Find a if a > b.

4. Find $\frac{8}{\pi}\sum_{k=1}^{\infty}\tan^{-1}\left(\frac{2k}{2+k^2+k^4}\right)$

5. Let $a_1, a_2, ..., a_n$ be an A.P. with common difference $\pi/6$ and assume
$\sec a_1 \sec a_2 + \sec a_2 \sec a_3 + ... + \sec a_{n-1} \sec a_n. = k (\tan a_n - \tan a_1)$
find the value of k.

6. If the lengths of the sides of a right angled triangle ABC right angled at C are in A.P., find 5 (sin A + sin B).

7. First term of an A.P. of non-constant terms is 3 and its second, tenth and thirty-fourth terms form a G.P., find the common difference.

8. A ball is dropped from a height of 900 cm. Each time it rebounds, it rises to 2/3 of the height it has fallen through. Find the two times of total distance travelled by the ball before it comes to rest in deca meters.

9. Find the largest positive term of the A.P. whose first two terms are 2/5 and 12/23.

10. If $\log_x y$, $\log_z x$, $\log_y z$ are in G.P. , xyz = 64 and x^3, y^3, z^3 are in A.P., find x + y + z.

11. Let S_{50}= 1 + (2) (3) + 4 + (5) (6) + 7 + (8) (9) + ... up to 50 terms then find $\frac{S_{50}}{25} - 1980$

12. If a_n denotes the coefficient of x^n in

$P(x) = (1 + x + 2x^2 + ... + 25x^{25})^2$, find $\frac{a_5}{5}$.

13. If the m^{th}, n^{rh} and p^{th} terms of an A.P. and G.P. are equal and are x, y, z then x^{y-z}, y^{z-x}, z^{x-y} is equal to

14. The intreior angles of a polygon are in A.P. the smallest angle is 120^0 and the common difference is 5^0, Find the number of sides of the polygon.

15. If total number of runs scored in n matches is $\left(\frac{n+1}{4}\right)(2^{n+1}-n-2)$ where n > 1, and the runs scored in the k^{th} match are given by k. 2^{n+1-k}, where $1 \le k \le n$. find n

SUBJECTIVE QUESTION

1. If n is a natural number such that $n = p_1^{\alpha_1}, p_2^{\alpha_2}.p_3^{\alpha_3}.....p_k^{\alpha_k}$ and $p_1, p_2, ...p_k$ are distinct primes, then show that $\log n \geq k\log 2$

2. Let p be the first of the n arithmetic means between two numbers and q the first of n harmonic means between the same numbers. Show that q does not lie between p and $\left(\frac{n+1}{n-1}\right)^2 p$

3. Let a_1, a_2, ... be positive real number in G.P. for each n, let A_n, G_n, H_n be respectively, the arithmetic mean, geometric mean , harmonic mean of a_1 , a_2 .....a_n. Find an expression for the geometric mean of G_1, G_2, ...G_n in terms of A_1, A_2, ...A_n , H_1, H_2, ...H_n.

4. If a, b, c are positive numbers, then prove that $(1+a)^7(1+b)^7(1+c)^7 > 7^7 a^4b^4c^4$

5. If $S_1, S_2, S_3, ...S_q$ are the sums of n term of q A.P. 's whose first terms are 1,2,3,.... q and common differences are 1,3,5,.... (2q-1) respectively, show that

$$S_1 + S_2 + S_3 + ... + S_q = \frac{1}{2}nq(nq+1)$$

6. A number consists of three digits in G.P. the sum of the right hand and left hand digits exceeds twice the middle digit by 1 and the sum of the left hand and middle digits is two third of the sum of the middle and right hand digits. Find the numbers.

7. Three number are in G.P. whose sum is 70. If the extremes be each multipled by 4 and the middle by 5, they will be in A.P. Find the numbers.

8. If S_1, S_2, S_3 denote the sum of n terms of 3 arithmetic series whose first terms are unity and their common difference are in H.P., Prove that $n = \frac{2s_3s_1 - s_1s_2 - s_2s_3}{s_1 - 2s_2 + s_3}$

9. Suppose a is a fixed real number such that $\frac{a-x}{px} = \frac{a-y}{qy} = \frac{a-z}{rz}$; If p,q,r are in A.P. prove that x, y, z are in H.P.

10. Solve the following equations for x and y .If $\log_{10} x + \frac{1}{2}\log_{10} x + \frac{1}{4}\log_{10} x + ... = y$ and

$$\frac{1+3+5+....+(2y-1)}{4+7+10+....+(3y+1)} = \frac{20}{7\log_{10} x}$$

11. Two consecutive numbers from 1,2,3....n are removed and Arithmetic mean of the remaining numbers is $\frac{105}{4}$. find n and those removed numbers.

12. Let S_k be the sum of the first k terms of an A.P. What must be the relation for the ratio $\frac{S_{kx}}{S_x}$ to be independent of x?

13. Given that a, b, c, α, β, γ are all positive quantities and $a\alpha, b\beta, c\gamma$ are all distinct, if a, b, c are in A.P. , α, β, γ are in H.P. and $a\alpha, b\beta, c\gamma$ are in G.P. prove that $a : b : c = \frac{1}{\gamma} : \frac{1}{\beta} : \frac{1}{\alpha}$

14. Find the sum of the infinitely decreasing G.P. whose third term, the triple product of the first term by the fourth term and the second term form, in the indicated order, an A.P. with the common difference equal to 1/8.

15. On the ground n stones are placed. The distance between the first and the second is one metre between second and third is three metre, between third and fourth is five metres and so. on. How for will a person have to travel to bring them one by one to the basket placed at the first stone if the start to from the basket.

PREVIOUS IIT-JEE

Single correct

1. If a, b, c, d are positive real numbers such that $a+b+c+d=2$, then $M=(a+b)(c+d)$ satisfie the relation **[IIT 1998]**

(A) $0<M\le 1$ (B) $1\le M\le 2$

(C) $2\le M\le 3$ (D) $3\le M\le 4$

2. Let $a_1,a_2,.........a_{10},$ be in A.P. and $h_1,h_2,.....h_{10}$ be in H.P. If $a_1=h_1=2$ and $a_{10}=h_{10}=3$ then $a_4h_7,$ is **[IIT 1999]**

(A) 2 (B)3

(C) 5 (D) 6

3. Consider an infinite geometric series with first term 'a' and common ratio r. If the sum is 4 and the second term is 3/4, then **[IIT 2000]**

(A) $a=\frac{7}{4},r=\frac{3}{7}$ (B) $a=2,r=\frac{3}{8}$

(C) $a=\frac{3}{2},r=\frac{1}{2}$ (D) $a=3,r=\frac{1}{4}$

4. If the sum of the first 2n terms of the A.P. 2,5,8,....... is equal to the sum of the first n term of the A.P. 57,59,61,........... , the n equals **[IIT 2001]**

(A) 10 (B) 12

(C) 11 (B) 13

5. Let the positive numbers a, b ,c, d be in A.P. Then abc, abd, acd and bcd are **[IIT 2001]**

(A) Not in A.P. /G.P. /H.P (B) in A.P.

(C) in G.P. (B) H.P.

6. The number of solutions of $\log_4(x-1)=\log_2(x-3)$ is **[IIT 2001]**

(A) 3 (B) 1

(C) 2 (B) 0

7. Suppose a, b,c are in A.P. a^2,b^2,c^2 are in G.P. If a < b < c and $a+b+c=\frac{3}{2}$, then the value of a is **[IIT 2002]**

(A) $\frac{1}{2\sqrt{2}}$ (B) $\frac{1}{2\sqrt{3}}$

(C) $\frac{1}{2}-\frac{1}{\sqrt{3}}$ (D) $\frac{1}{2}-\frac{1}{\sqrt{2}}$

8. An infinite G.P. has first term ‘x’ and sum ‘5’, then x belongs to **[IIT 2004]**

(A) $x<-10$ (B) $-10<x<0$

(C) $0<x<10$ (D) $x>10$

Multiple Choice questions with more than one

9. Indicate the correct alternative (s), for $0<\phi<\pi/2$, if

$x=\sum_{n=0}^{\infty}\cos^{2n}\phi$, $y=\sum_{n=0}^{\infty}\sin^{2n}\phi$, $z=\sum_{n=0}^{\infty}\cos^{2n}\phi\sin^{2n}\phi$ then **[IIT 1993]**

(A) xyz = xz+ y (B) xyz = xy +z

(C) xyz = x+y+z (D) xyz = yz +x

10. Let T_r be the r^{th} term of an AP, for r = 1,2,3.......... if for some positive integers m, n we have $T_m=\frac{1}{n}$ and $T_n=\frac{1}{m}$, then T_{mn} equals **[IIT 1998]**

(A) $\frac{1}{mn}$ (B) $\frac{1}{m}+\frac{1}{n}$

(C) 1 (D) 0

11. If $x>1,y>1,z>1$ are in GP, then $\frac{1}{1+\ell nx},\frac{1}{1+\ell ny},\frac{1}{1+\ell nz}$ are in **[IIT 1998]**

(A) AP (B) HP

(C) GP (D) None of these

12. A straight line through the vertex P of a triangle PQR intersects the side QR at the point S and the circumcircle of the triangle PQR at the point T. If S is not the centre of the circumcircle, then **[IIT 2008]**

(A) $\frac{1}{PS}+\frac{1}{ST}<\frac{2}{\sqrt{QS\times SR}}$ (B) $\frac{1}{PS}+\frac{1}{ST}>\frac{2}{\sqrt{QS\times SR}}$

(C) $\frac{1}{PS}+\frac{1}{ST}<\frac{4}{QR}$ (D) $\frac{1}{PS}+\frac{1}{ST}>\frac{4}{QR}$

13. Let $S_n=\sum_{k=1}^{n}\frac{n}{n^2+kn+k^2}$ and $T_n=\sum_{k=0}^{n-1}\frac{n}{n^2+kn+k^2}$ for n = 1,2,3,......... Then, **[IIT 2008]**

(A) $S_n<\frac{\pi}{3\sqrt{3}}$ (B) $S_n>\frac{\pi}{3\sqrt{3}}$

(C) $T_n<\frac{\pi}{3\sqrt{3}}$ (D) $T_n>\frac{\pi}{3\sqrt{3}}$

Comprehension Passage

I. Let V_r denote the sum of the first r terms of an arithmetic progression (A.P.) whose first term is r and the common difference is $(2r-1)$ Let

$T_r = V_{r+1} - V_r - 2$ and $Q_r = T_{r+1} - T_r$ for r = 1, 2,... **[IIT 2007]**

14. The sum $V_1 + V_2 + ... + V_n$ is

(A) $\frac{1}{12}n(n+1)(3n^2 - n + 1)$ (B) $\frac{1}{12}n(n+1)(3n^2 + n + 2)$

(C) $\frac{1}{2}n(2n^2 - n + 1)$ (D) $\frac{1}{3}(2n^3 - 2n + 3)$

15. T_r is always

(A) an odd number (B) An even number

(C) A prime number (D) A composite number

16. Which one of the following is a correct statement?

(A) $Q_1, Q_2, Q_3, ...$ are in A.P. with common difference 5

(B) $Q_1, Q_2, Q_3, ...$ are in A.P. with common difference 6

(C) $Q_1, Q_2, Q_3, ...$ are in A.P. with common difference 11

(D) $Q_1 = Q_2 = Q_3 = ...$

II. Let A_1, G_1, H_1 denote the arithmetic, geometric and harmonic means, respectively, of two distinct positive number. For $n \geq 2$, let A_{n-1} and H_{n-1} have arithmetic, geometric and harmonic means as A_n, G_n, H_n respectively. **[IIT 2007]**

17. Which one of the following statements is correct?

(A) $G_1 > G_2 > G_3 > ....$ (B) $G_1 < G_2 < G_3 < ....$

(C) $G_1 = G_2 = G_3 = ...$ (D) $G_1 < G_3 < G_5 < ....$ and $G_2 > G_4 > G_6 > ....$

18. Which one of the following statements is correct?

(A) $A_1 > A_2 > A_3 > ...$ (B) $A_1 < A_2 < A_3 <$

(C) $A_1 > A_3 > A_5 > ....$ and $A_2 < A_4 < A_6 < ...$

(D) $A_1 < A_3 < A_5 < ...$ and $A_2 > A_4 > A_6 > ...$

19. Which one of the following statements is correct?

(A) $H_1 > H_2 > H_3 > ...$ (B) $H_1 < H_2 < H_3 < ....$

(C) $H_1 > H_3 > H_5 > ....$ and $H_2 < H_4 < H_6 < ...$

(D) $H_1 < H_3 < H_5 < H_2 > H_4 > H_6 > ...$

(Assertion – Reason Type)

Each question contains STATEMENT – 1 (Assertion) and STATEMENT – 2 (Reason). Each question has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

20. Suppose four distinct positive numbers a_1, a_2, a_3, a_4 are in G.P. Let $b_1 = a_1, b_2 = b_1 + a_2, b_3 = b_2 + a_3$ and $b_4 = b_3 + a_4$.

STATEMENT 1 : The numbers $a_1, b_2, b_3, b4$ are neither in A.P. nor in G.P.

STATEMENT 2 :The numbers b_1, b_2, b_3, b_4 are in H.P. **[IIT 2008]**

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True; Statement–2 NOT a correct explanation for Statement-1.

(C) Statement–1 is True, Statement–2 is False

(D) Statement –1 is False, statement–2 is True.

Subjective

21. If p be the first of n arithmetic means between two numbers and q be the first of n harmonic means between the same two numbers, prove that the value of q cannot be between p and $\left(\frac{n+1}{n-1}\right)^2$ p. **[IIT1991]**

22. If $S_1, S_2, S_3, ...S_n$ are the sums of infinite geometric series whose first terms are 1,2,3,..., n and whose common rations are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, ... \frac{1}{n+1}$ respectively, then find the value of $S_1^2 + S_2^2 + S_3^2 + ... + S_{2n-1}^2$. **[IIT1991]**

23. The real number x_1, x_2, x_3 satisfying the equation $x^3 + x^2 + \beta x + \gamma = 0$ are in A.P. Find the intervals in which β *and* γ lie. **[IIT1996]**

24. The fourth power of the common difference of an arithmetic progression with integer entries added to the product of any four consecutive terms of it. Prove that the resulting sum is the square of an integer. **[IIT1999]**

25. Let $a_1, a_2, ...a_n$ be positive real numbers in G.P. for each n, let A_n, G_n, H_n, be respectively, the arithmetic mean, geometric mean and harmonic mean of $a_1, a_2, a_3, ...a_n$. Find an expression for the G.M. of $G_1, G_2, ..G_n$ in terms of $A_1, A_2, ...A_n$, $H_1, H_2, ...H_n$.

[IIT 2001]

26. Let a, b be positive real numbers, If a, A_1, A_2, b are in arithmetic progression a, G_1, G_2, b are geometric progression and a, H_1, H_2, b are in harmonic progression, show that $\frac{G_1G_2}{H_1H_2} = \frac{A_1 + A_2}{H_1 + H_2} = \frac{(2a+b)(a+2b)}{9ab}$. **[IIT 2002]**

27. If a, b,c are in A.P. a^2, b^2, c^2 are in H.P. Then prove that either $a = b = c$ or $a, b, -c/2$ form a G.P. **[IIT 2003]**

28. Prove that $(a+1)^7(b+1)^7(c+1)^7 > 7^7 a^4b^4c^4$, where $a, b, c \in R^+$. **[IIT 2004]**

29. For n= 1,2,3,......., Let $A_n = \frac{3}{4} - \left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^3 - ..... + (-1)^{n-1}\left(\frac{3}{4}\right)^n$, and $B_n = 1 - A_n$. Find the smallest natural number n_0 such that $B_n > A_n$ for all $n \geq n_0$. **[IIT 2006]**

Integer Type

30. Let S_k, k = 1, 2, ... , 100, denote the sum of the infinite geometric series whose whose first term is $\frac{k-1}{k!}$ and the common ratio is $\frac{1}{k}$. Then the value of $\frac{100^2}{100!} + \sum_{k=1}^{100} \left|(k^2 - 3k + 1)S_k\right|$ is

[IIT 2010]

31. Let $a_1, a_2, a_3, ... , a_{11}$ be real numbers satisfying $a_1 = 15$, $27 - 2a_2 > 0$ and $a_k = 2a_{k-1} - a_{k-2}$ for k = 3, 4, ... , 11. If $\frac{a_1^2 + a_2^2 + ... + a_{11}^2}{11} = 90$, then the value of $\frac{a_1 + a_2 + ... + a_{11}}{11}$ is equal to

[IIT 2010]