Progression and Series: Answer

ANSWER KEY

CONCEPTUAL QUESTIONS

1. A	2. B	3. B)	4. A	5. A	6. D	7. A
8. B	9.A	10. A				

Objective questions (One correct Answer)

LEVEL -I

1. C	2. D	3. D	4. A	5. D	6. B	7. A
8. A	9. B	10. B	11. C	12. C	13. A	14. B
15. C	16. A	17. C	18. A	19. B	20. C	21. C
22. A	23. A	24. A	25. A			

LEVEL -II

26. B	27. D	28. B	29. A	30. C	31. C	32. B
33. C	34. C	35. D	36. B	37. B	38. B	39. B
40. C	41. D	42. C	43. B	44. C	45. B	46. B
47. C	48. A	49. C	50. C			

LEVEL -III

51. A	52. B	53. B	54. B	55. A	56. A	57. D
58. D	59. B	60. C				

ONE OR MORE THAN ONE CORRECT ANSWERS

LEVEL -I

1. A,C	2. A,B,C,D	3. A,B,C,D	4. B,D	5. A,B,C,D
6. A,C	7. B,C	8. A,C	9. B,D	10. A,D
11. A,B	12. B,D,	13. C,D	14. ,A,B,C,D	15. A,B,C
16. A,D	17. A,C	18. A,D	19. B,C,D	20. A,D

LEVEL -II

21. A,B,C	22. A,D	23. A,D	24. B,D	25. A,C,D
26. A,D	27. A,B	28. A,D	29. B,C	30. A,B,C,D

LEVEL -III

31. A,B	32. B,C	33. A,B,C,D	34. A,B,C,D	35. B,C

LINKED COMPREHENSION TYPE

1. D	2. A	3. B	4. C	5. B
6. B	7. A	8. B	9. A	10. B
11. C	12. A			

Matrix–Match Type

1. (A-s), (B-r,t) (C-q,t), (D-p)
2. (A-p), (B-q), (C-r), (D-s)
3. (A-s), (B-r), (C-q), (D-p)
4. (A-r,t), (B-s), (C-p,t), (D-q,t)

ASSERTION – REASON TYPE

1. A	2. C	3. C	4. A	5. B
6. A	7. A	8. B	9. C	10. A

INTEGER ANSWER TYPE QUESTIONS

1. 3	2. 4	3. 9	4. 2	5. 2
6. 7	7. 1	8. 9	9. 5	10. 12
11. 7	12. 6	13. 1	14. 9 or 16	15. 7

SUBJECTIVE QUESTION

6. 469
7. 10, 20, 40 or 40, 20, 10
10. $x = 10^5$, $y = 10$
14. 2
15. $(n - 1)(n)(2n - 1)$

Previous IIT. JEE
Single correct

1. A	2. D	3. D	4. C	5. D
6. D	7. D	8. C		

MULTIPLE CHOICE QUESTIONS WITH ONE OR MORE THAN ONE CORRECT ANSWERS:

9. B,C 10. C 11. B 12. B,D 13. A

LINKED COMPREHENSION TYPE

14. B 15. D 16. B 17. C 18. A 19. B

ASSERTION – REASON TYPE

20-C

SUBJECTIVE QUESTION

22. $\frac{n(2n+1)(4n+1)-3}{3}$

23: $\beta \in \left(-\infty, \frac{1}{3}\right], \gamma \in \left[-\frac{1}{27}, \infty\right)$

25. $\left[(A_1, A_2, ...A_n)(H_1, H_2 ....H_n)\right]^{\frac{1}{2n}}$

29. Least value of $n_0 = 6$

INTEGER TYPE

30. (4)
31. (0)

HINTS & SOLUTIONS

Conceptual Questions

1. Let x be the first term and d be the c. d of A.P.

a = x + (p - 1) d

b = x+ (q-1) d

$$\Rightarrow d = \frac{a-b}{p-q} \quad \text{----------(1)}$$

$$\text{so, } x = a - \frac{(p-1)(a-b)}{p-q}$$

$$= \frac{pa - qa - pa + pb + a - b}{p-q} = \frac{pb - qa + a - b}{p-q}$$

$$\text{Hence, } s_{p+q} = \frac{p+q}{2}\left[a + b + \frac{a-b}{p-q}\right]$$

2. Let first term = a c. d = d, S_m = sum of first m terms

Then given $S_m = S_{m+n} - S_m = S_{m+p} - S_m$

$$S_m = S_{m+n} - S_m \Rightarrow 2S_m = S_{m+n}$$

$$\Rightarrow 2\frac{m}{2}\left(2a + (m-1)d\right) = \frac{m+n}{2}\left[2a + (m+n-1)d\right]$$

$$2a\left[2m - m - n\right] = d\left[m^2 + n^2 + 2mn - m - n - 2m^2 + 2m\right]$$

$$2a\left[m-n\right] = d\left[n^2 - m^2 + 2mn + m - n\right] \quad \text{....... (i)}$$

also 2 $S_m = S_{m+p}$

$$\Rightarrow 2a\left[m-p\right] = d\left[p^2 - m^2 + 2mp + m - p\right] \quad \text{....... (ii)}$$

from (i) and (ii)

$$\Rightarrow \frac{n^2 - m^2 + 2mn + m - n}{m-n} = \frac{p^2 - m^2 + 2mp + m - p}{m-p}$$

$$\Rightarrow -(n+m) + 1 + \frac{2mn}{m-n} = -(p+m) + 1 + \frac{2mp}{m-p}$$

$$\Rightarrow (m+n)\frac{(m-p)}{mp} = (m+p)\frac{(m-n)}{mn}$$

$$\Rightarrow (m+n)\left(\frac{1}{m} - \frac{1}{p}\right) = (m+p)\left(\frac{1}{m} - \frac{1}{n}\right)$$

$$\Rightarrow \left(\frac{n+m}{n-m}\right)\left(\frac{p-m}{p+m}\right) = \frac{p}{n}$$

3. We have, $S_k = a + ar + ... + ar^{k-1} = \dfrac{a\left(1-r^k\right)}{1-r}$

$\therefore$ $S_1 + S_2 + S_3 + ... + S_n = \dfrac{na}{1-r} - \dfrac{a}{1-r}(r + r^2 + ... + r^n)$

$$= \frac{na}{1-r} - \frac{a}{1-r}\frac{r\left(1-r^n\right)}{1-r} = \frac{na}{1-r} - \frac{ar\left(1-r^n\right)}{\left(1-r\right)^2}$$

4. The given sum S = $(x + y) + (x^2 + xy + y^2) + ...$

$$= \frac{1}{(x-y)}\left\{\left(x^2-y^2\right)+\left(x^3-y^3\right)+\left(x^4-y^4\right)+....\right\}$$

$$= \frac{1}{(x-y)}\left\{\left(x^2+x^3+....\right)-\left(y^2+y^3+....\right)\right\}$$

$$= \frac{1}{(x-y)}\left(\frac{x^2}{1-x}-\frac{y^2}{1-y}\right) = \frac{1}{x-y}\left(\frac{x^2-y^2-x^2y+xy^2}{(1-x)(1-y)}\right) = \frac{x+y-xy}{(1-x)(1-y)}$$

5. Given $b^2 = ac, x = \dfrac{a+b}{2}, y = \dfrac{b+c}{2}$

Consider $\dfrac{a}{x}+\dfrac{c}{y} = \dfrac{2a}{(a+b)}+\dfrac{2c}{(b+c)} = \dfrac{2a}{a+\sqrt{ac}}+\dfrac{2c}{\sqrt{ac}+c}$

$$= 2\left(\frac{\sqrt{a}+\sqrt{c}}{\sqrt{a}+\sqrt{c}}\right) = 2$$

6. Number of students giving wrong answers to at least r questions = 2^{p-r}
Number of students giving wrong answers to at least (r + 1) questions = 2^{p-r-1}
$\therefore$ Number of students giving wrong answers to exactly r questions
= $2^{p-r} - 2^{p-r-1}$. Also number of students giving wrong answers to exactly p questions
= $2^{p-p} = 2^0 = 1$
$\therefore$ Total number of wrong answers

$$1\left(2^{p-1}-2^{p-2}\right)+2\left(2^{p-z}-2^{p-3}\right)+.........+(p-1)\left(2^1-2^0\right)+n\left(2^0\right)$$

$$= 2^{p-1}+\left(-2^{p-2}+2.2^{p-2}\right)+\left(-2.2^{p-3}+3.2^{p-3}\right)+........+\left\{-(n-1)2^0+n.2^0\right\}$$

$= 2^{p-1} + 2^{p-2} + 2^{p-3} + ... + 2^0 = 2^p - 1$

$$\Rightarrow 2^p - 1 = 2047 \Rightarrow 2^p = 2048 = 2^{11} \Rightarrow p = 11$$

7. Since $a, b, c \in R^+$ and distinct $\Rightarrow AM > GM > HM$

Since $b = \sqrt{ac}$ and consider AM and HM of a and c,

$\Rightarrow \frac{a+c}{2} > b > \frac{2ac}{a+c}$

From first inequality (a + c) > 2b $\Rightarrow a^2 + ac - 2ab > 0$

From second inequality b (a + c) > 2ac

$\Rightarrow 2ab + 2bc - 4ac > 0$

Adding the two inequalities $a^2 + 2bc - 3ac > 0$

8. Consider the opposite numbers

a^x, a^x, ... ky times and b^y, b^y, ... kx times

$$AM = \frac{\{a^x + a^x + ...ky \text{ times}\} + \{b^y + b^y + ...kx \text{ times}\}}{kx + ky} = \frac{kya^x + kxb^y}{k(x+y)} = \frac{ya^x + xb^y}{(x+y)}$$

$$GM = \left\{\left(a^x.a^x......ky \text{ times}\right)\left(b^y.b^y........kx \text{ times}\right)\right\}^{\frac{1}{k(x+y)}}$$

$$= \left(a^{x(ky)}.b^{y(kx)}\right)^{\frac{1}{k(x+y)}} (ab)^{\frac{kxy}{k(x+y)}} = (ab)^{\frac{xy}{(x+y)}} \qquad .... (i)$$

As $\frac{1}{x} + \frac{1}{y} = 1, \frac{x+y}{xy} = 1$, i.e., $x + y = xy$

$\therefore (i)$ becomes $\frac{ya^x + xb^y}{xy} \geq ab$ or $\frac{a^x}{x} + \frac{b^y}{y} \geq ab$

9. Keeping in view that a + b + c = 18

$a^2b^3c^4$ is max. when $\left(\frac{a}{2}\right)^2\left(\frac{b}{3}\right)^3\left(\frac{c}{4}\right)^4$ is max.

The sum of factors is $2.\frac{a}{2} + 3.\frac{b}{3} + 4.\frac{c}{4}$ = a + b + c = 18

hence product will be max. when all the factors are equal $\frac{a}{2} = \frac{b}{3} = \frac{c}{4} = \frac{a+b+c}{2+3+4} = \frac{18}{9} = 2$

$\therefore$ a = 4, b = 6, c = 8

$\therefore$ Max. value of $a^2 b^3 c^4$ is $4^2 6^3 8^4$.

10. Let 1st term is x and $(2n-1)^{th}$ term = y

1. In $A.P.$ $t_n = \frac{x+y}{z} = a$

2. in G.P $t_n = \sqrt{xy} = b$

H.P. $t_n = \frac{2xy}{x+y} = c$

$\Rightarrow b^2 = ac \text{ and } a > b > c$, equality holds good if $a = b = c$

Objective questions (One correct Answer)
Level-I

1. $S_{2n} = S_n'$

$\Rightarrow \frac{2n}{2}\left[2.2+(2n-1)3\right]=\frac{n}{2}\left[2.57+(n-1)2\right]$

$\Rightarrow 5n = 55$

2. $S=\frac{1}{1-(1/2)}=2$

$S_n=\frac{1-(1/2)^n}{1-(1/2)}=\frac{1}{2^{n-1}}$ =2- $\frac{1}{2^{n-1}}$

$S-S_n=\frac{1}{2^{n-1}}<\frac{1}{1000}$ or $2^{n-1}\geq 1000$

Now 2^{10} = 32 × 32 = 1024

$\therefore$ n – 1 $\geq$ 10 or n $\geq$ 11

Hence the least value is 11.

3. We given, $a_5+a_{20}=a_1+a_{24}, a_{10}+a_{15}=a_1+a_{24}$

Hence the given relations reduce to, $3(a_{1,}+a_{24})=225$, giving $a_1+a_{24}=75$

Hence S_{24} = $\frac{n}{2}(a+l)=(24/2)(a_1+a_{24})=12\times 75=900$

4. $S_p=\frac{p}{2}[2A+(p-1)d]=a$

$\therefore \frac{2a}{p}=2A+(p-1)d$ .... (i)

$\therefore \frac{2b}{q}=2A+(q-1)d$ .... (ii)

$\therefore \frac{2c}{r}=2A+(r-1)d$ .... (iii)

Multiply (i), (ii) and (iii) by q – r, r – p and p – q respectively and add

$\therefore \Sigma \frac{a}{p}(q-r)=0$

5. Since the numbers are in A.P.

$\therefore$ 28 = $3^{2\sin 2\theta-1}$ + $3^{4-2\sin 2\theta}$

or $28=\frac{9^{\sin 2\theta}}{3}+\frac{81}{9^{\sin 2\theta}}$, where x = $9^{\sin 2\theta}$

or $x^2-84x+243=0$

or (2.81) (x – 3) = 0 $\quad \therefore x=81 \text{ or } 3$

$\therefore x=9^{\sin 2\theta}=81, 3 \text{ or } 9^2, 9^{1/2}$

$\therefore \sin 2\theta = 2 \text{ or } 1/2$

since sin 2θ cannot be greater than 1 so we choose sin $2\theta = \frac{1}{2}$

Hence the terms in A.P. are

3^0 , 14, 27 i.e. 1, 14, 27.

$\therefore T_5 = a + 4d$ = 1 + 4.13 = 53

6. $T_m = S_m - S_{m-1}$

$\therefore$ **164 = 3 (2m – 1) + 5.1 $\therefore$ 6m = 162**

7. We can rewrite the series as

$$1 + 1 + \frac{5}{x} + \left(\frac{5}{x}\right)^2 + \left(\frac{5}{x}\right)^3 + ...$$

We can sum up this series if |5/x| < 1

$\Leftrightarrow |x| > 5$

8. $b = \frac{2ac}{a+c}$ and $c = \frac{2bd}{b+d}$

$$\therefore (a+c)(b+d) = \frac{2ac}{b}.\frac{2bd}{c} = 4ad$$

$\Rightarrow ab + bc + cd = 3ad$

9. L.C.M. of 2 and 5 is 10.

Numbers divisible by 2 will contain numbers which are also divisible by 10. Similarly numbers divisible by 5 will contain numbers which are also divisible by 10. Thus the number divisible by 10 will occur twice. Hence we can

write S = $S_2 + S_5 - S_{10}$

Now, $S_2 = \frac{2.50.51}{2} = 2550$ by $\Sigma n = \frac{n(n+1)}{2}$

Similarly, $S_5 = 1050$, $S_{10} = 550$

$\therefore$ S = $S_2 + S_5 - S_{10}$

= 2550 + 1050 – 550 = 3050

10. $S_n = (2n-4)\frac{\pi}{2} = (n-2)180^0$ (formula for polygon)

a = 120^0, d = 5^0

$S_n = \frac{n}{2}[2a + (n-1)d]$ for A.P.

$\therefore n^2 - 25n + 144 = 0$

$\therefore$ n = 9, 16

But when n = 16 then $T_{16} = 195^0$ which is not possible

$\therefore$ n = 9 only

11. $a_1, a_2, a_3 .... a_n$ are in H.P. $\frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3} ...... \frac{1}{a_n}$ are in A.P.

Multiply each term by $a_1 + a_2 + a_3 + ... + a_n$ then substract 1 from each term

we get $\frac{a_2 + a_3 + ... + a_n}{a_1}, \frac{a_1 + a_3 + ... + a_n}{a_2}, ... \frac{a_1 + a_2 + ... + a_{n-1}}{a_n}$ are in A.P.

$\therefore \frac{a_1}{a_2 + a_3 + ... + a_n}, \frac{a_2}{a_1 + a_3 + ... + a_n}, ... \frac{a_n}{a_1 + a_2 + ... + a_{n-1}}$ in H.P.

12. We can write the given equation as

$$\log_2\left(x^{1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+..........}\right)=4$$

$$\Rightarrow \log_2\left(x^2\right)=4 \Rightarrow x^2=2^4 \Rightarrow x=4$$

13. $\frac{S_m}{S_n}=\frac{m^2}{n^2}$ $\therefore \frac{S_m}{m^2}=\frac{S_n}{n^2}=k$ (say)

$$\frac{T_m}{T_n}=\frac{S_m-S_{m-1}}{S_n-S_{n-1}}=\frac{k\left[m^2-(m-1)^2\right]}{k\left[n^2-(n-1)^2\right]}=\frac{2m-1}{2n-1}$$

14. $\frac{S_n}{S'_n}=\frac{(n/2)[2a+(n-1)d]}{(n/2)\left[2a'+(n-1)d'\right]}$

$=\frac{3n+8}{7n+15}$

or, $\frac{a+\left(\frac{n-1}{2}\right)d}{a'+\left(\frac{n-1}{2}\right)d'}=\frac{3n+8}{7n+15}$ ...(i)

We have to find $\frac{T_{12}}{T'_{12}}=\frac{a+11d}{a'+11d'}$

Choosing (n – 1) / 2 = 11 or n = 23 in (i)

we get, $\frac{T_{12}}{T'_{12}}=\frac{7}{16}$

15. If the numbers by x, y, z then
$1/x = \log_2 3, 1/y = \log_2 2.3 = 1 + \log_2 3$,
$1/z = \log_2 (4 \times 3) = 2 + \log_2 3$ which are in A.P.
$\therefore$ x, y, z are in H.P.

16. $T_{m+n}=ar^{m+n-1}=p$ $\quad$ $T_{m-n}=ar^{m-n-1}=q$

Multiplying $a^2r^{2m-2}=pq$

$\therefore T_m=ar^{m-1}=\sqrt{(pq)}$

17. $1, x_1, x_2 ... x_m$, 31 is an A.P. of (m + 2) terms.
$31 = T_{m+2} = a + (m + 1)d = 1 + (m + 1)d$

$\therefore d=\frac{30}{(m+1)}$

Now $\frac{x_7}{x_{m-1}}=\frac{5}{9}$ $\quad \therefore \frac{T_8}{T_m}=\frac{a+7d}{a+(m-1)d}=\frac{5}{9}$

Now put for a and d and we get m = 14.

18. We have

$$\frac{\pi}{4}=\left(1-\frac{1}{3}\right)+\left(\frac{1}{5}-\frac{1}{7}\right)+\left(\frac{1}{9}-\frac{1}{11}\right)+.........$$

$$= \frac{2}{1.3} + \frac{2}{5.7} + \frac{2}{9.11} + ...$$

$$\Rightarrow \frac{1}{1.3} + \frac{1}{5.7} + \frac{1}{9.11} + ... = \frac{\pi}{8}$$

19. If t_r denotes the nth term of the series, then

$$xt_r = \frac{x}{(1+rx)(1+(r+1)x)} = \frac{1}{1+rx} - \frac{1}{1+(r+1)x}$$

$$\Rightarrow x\sum_{r=1}^{n} t_r = \sum_{r=1}^{n}\left[\frac{1}{1+rx} - \frac{1}{1+(r+1)x}\right]$$

$$= \frac{1}{1+x} - \frac{1}{1+(n+1)x} = \frac{nx}{(1+x)(1+(n+1)x)}$$

$$\Rightarrow \sum_{r=1}^{n} t_r = \frac{n}{(1+x)[1+(n+1)x]}$$

20. We have

$$\frac{1}{k}(1+2+3+........+k) = \frac{1}{k}\frac{k(k+1)}{2} = \frac{k+1}{2}$$

Thus, $S = \frac{1}{2}[2+3+4+.........21] = \frac{10}{2}(2+21) = 115$

21. We have $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + ...$ upto ∞

$$= \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \frac{1}{6^2} ....$$ upto ∞

$$-\frac{1}{2^2}\left[1 + \frac{1}{2^2} + \frac{1}{3^2} + ...\right]$$

$$= \frac{\pi^2}{6} - \frac{1}{4}\left(\frac{\pi^2}{6}\right) = \frac{\pi^2}{8}$$

22. We can write S as
S = (1 – 2) (1 + 2) + (3 – 4) (3 + 4) + ... + (2001 – 2002) (2001 + 2002) + 2003^2
= [1 + 2 + 3 + 4 + ... + 2002] + 2003^2

= $-\frac{1}{2}(2002)(2003) + 2003^2 = 2007006$

23. Using $AM \geq GM$

$$\frac{a+b+c}{3} \geq (abc)^{\frac{1}{3}}$$

$\frac{3b}{3} \geq (abc)^{\frac{1}{3}}$ (since 2b = a + c)

$$b \geq 4^{\frac{1}{3}}$$

24. Since $AM \geq HM$

$$\frac{x+y+z}{3} \geq \frac{3}{\frac{1}{x}+\frac{1}{y}+\frac{1}{z}}$$

$$\Rightarrow \frac{a}{3} \geq \frac{3}{\frac{1}{x}+\frac{1}{y}+\frac{1}{z}} \text{ or } \frac{1}{x}+\frac{1}{y}+\frac{1}{z} \geq \frac{9}{a}$$

25. Since a and b are unequal, $\frac{a^2+b^2}{2} > \sqrt{a^2b^2}$

(A.M. > G.M. for unequal numbers)

$\Rightarrow a^2+b^2 > 2ab$

Similarly $b^2 + c^2 > 2bc$ and $c^2 + a^2 > 2ca$

Hence $2(a^2 + b^2 + c^2) > 2(ab + bc + ca)$

$\Rightarrow ab+bc+ca < 1$

LEVEL - II

26. $x+y+z=1 \Rightarrow 2.\frac{x}{2}+3.\frac{y}{3}+4.\frac{z}{4}=1$

using weighted mean

$$\frac{2.\left(\frac{x}{2}\right)+3.\left(\frac{y}{3}\right)+4.\left(\frac{z}{4}\right)}{9} \geq \left(\left(\frac{x}{2}\right)^2\left(\frac{y}{3}\right)^3\left(\frac{z}{4}\right)^4\right)^{\frac{1}{9}}$$

$$\left(\frac{1}{9}\right)^9 \geq \frac{x^2y^3z^4}{2^{10}3^3} \Rightarrow x^2y^3z^4 \leq \frac{2^{10}}{3^{15}}$$

27. $AM \geq GM$

$$\frac{\frac{b}{a}+\frac{c}{a}+\frac{c}{b}+\frac{a}{b}+\frac{a}{c}+\frac{b}{c}}{6}$$

$\geq (1)^{1/6}$

$\Rightarrow$ minimum value is 6

28. $S = a + ar + ar^2 + ar^2 + ... + ar^{n-1}$

i.e. n terms

$$S = \frac{a(1-r^n)}{(1-r)} \quad \text{..... (i)}$$

$\therefore$ P = product = $a.\ ar.\ ar^2....ar^{n-1}$

$= a^n r^{1+2+3+4+5+....n-1} = a^n.r^{n(n-1)/2}$

$\therefore p^2 = a^{2n}r^{n(n-1)}$ ..... (ii)

$$R = \frac{1}{a}+\frac{1}{ar}+\frac{1}{ar^2}+\frac{1}{ar^3}+....+\frac{1}{ar^{n-1}} \text{ (n terms)}$$

$$\therefore R = \frac{1}{a}\frac{\left(1-\frac{1}{r^n}\right)}{1-1/r} = \frac{(r^n-1)}{(r-1)}.\frac{1}{ar^{n-1}} \text{ ..... (iii)}$$

$\therefore \frac{S}{R} = a.\frac{(1-r^n)}{1-r}.\frac{(r-1)}{(r^n-1)}.ar^{n-1} = a^2r^{(n-1)}$ by (i) and (ii).

$\therefore [S/R]^n = a^{2n}r^{n(n-1)} = p^2$ by (ii)

29. $T_p = AR^{p-1} = x$

$\log x = \log A + (p-1)\log R$

Similary write log y , log z

Multiply by q – r, r – p and p – q and add we get,

$(q-r)\log x + (r-p)\log y + (p-q)\log z = 0$

30. $ar(1+r^3) = 216$ and $\frac{ar^3}{ar^5} = \frac{1}{4}$

$\Rightarrow r^2 = 4 \Rightarrow r = 2, -2$

when r = 2 then 2a (9) = 216 $\therefore a = 12$

when r = – 2, then – 2a (1 – 8) = 216

$\therefore a = \frac{216}{14} = \frac{108}{7}$, which is not an integer.

31. Sum of three numbers in A.P.

= 3a = 12

$\therefore (x-4)(x^2-8x+7) = 0$

$\therefore x = 1, 4, 7$ or $7, 4, 1$, $d = \pm 3$

32. $(a^2 + b^2 + c^2)(b^2 + c^2 + d^2) \leq (ab + bc + cd)^2$,

Solving this we get

$b^4 + c^4 + a^2c^2 + a^2b^2 + b^2d^2 + b^2c^2 - 2ab^2c - 2bc^2d - 2abcd \leq 0$

or $(b^2-ac)^2 + (c^2-bd)^2 + (ad-bc)^2 \leq 0$

$\therefore b^2 - ac = 0 \Rightarrow \frac{b}{a} = \frac{c}{b}$, $c^2 - bd = 0 \Rightarrow \frac{c}{b} = \frac{d}{c}$, $ab - bc = 0 \Rightarrow \frac{d}{c} = \frac{b}{a}$

$\therefore \frac{b}{a} = \frac{c}{b} = \frac{d}{c}$ Hence a, b, c, d are in G.P.

33. Let c.d = d

$a_p = a_1 + (p-1)d$, $a_q = a_1 + (q-1)d$, $a_r = a_1 + (r-1)d$

as a_p, a_q, a_r are in G.P.

$\frac{a_q}{a_p} = \frac{a_r}{a_q} = \frac{a_q - a_r}{a_p - a_q}$ (by law of proportions)

or $\frac{a_q}{a_p} = \frac{a_r}{a_q} = \frac{a_1 + (q-1)d - a_1 - (r-1)d}{a_1 + (p-1)d - a_1 - (q-1)d} = \frac{q-r}{p-q}$ or $\frac{a_q}{a_p} = \frac{q-r}{p-q} = \frac{r-q}{q-p}$

34. $a + ar + ar^2 = x.\ ar$

or, $r^2 + r(1-x) + 1 = 0$, r is real

$\Delta > 0$ i.e. $(1-x)^2 - 4 > 0$

or, $x^2 - 2x - 3 > 0$

or, $(x + 1)(x - 3) > 0$

$\Rightarrow x < -1$ or $x > 3$

35. $x \log a = y \log b = z \log c = k$ (say)

Also $y^2 = xz$

$$\frac{k^2}{(\log b)^2} = \frac{k^2}{\log a.\log c}$$

or $\frac{\log a}{\log b} = \frac{\log b}{\log c}$ or $\log_b a = \log_c b$

36. Let the common ratio be taken as k and a be the first term.

$R = T_r = ak^{r-1}$

$\therefore R^{s-t} = a^{s-t}k^{(r-1)(s-t)}$ similarly

$S^{t-r} = a^{t-r}k^{(s-1)(t-r)}$

$T^{r-s} = a^{r-s}k^{(t-1)(r-s)}$

Multiplying the above three and knowing that

$A^m.A^n.A^p = A^{m+n+p}$

$\therefore R^{s-t}S^{1-r}T^{r-s} = a^0.k^0 = 1$

$\because \Sigma(a-b) = 0, \Sigma(a+\lambda)(b-c) = 0$

37. Three numbers in G.P. are $\frac{a}{r}, a, ar$

then $\frac{a}{r}, 2a, ar$ are in A.P. as given

$\therefore 2(2a) = a\left(r + \frac{1}{r}\right)$

or, $r^2 - 4r + 1 = 0$ or $r = 2 \pm \sqrt{3}$

or, $r = 2 + \sqrt{3}$ as $r > 1$ for an increasing G.P.

38. Let numbers be a & b

$\Rightarrow a, g, b$ in G.P. and a, p, q, b in A.P.

$\Rightarrow g^2 = ab$ & $p - a = q - p = b - q$

we get $a = 2p - q$ & $b = 2q - p$

so $g^2 = (2p - q)(2q - p)$

39. Let T_1 & T_{2n+1} are A & B

Between there terms A + B; T_n = a, b, c when series are in A.P, G.P & H.P. respectively.

$\Rightarrow a = \frac{A+B}{2}, b = \sqrt{AB}, c = \frac{2AB}{A+B}$

$\Rightarrow b^2 = ac$

40. Let the two numbers a and b

given $a + b = \frac{13}{6}$

and A.M.' s are $A_1, A_2, \ldots\ldots\ldots A_{2n}$ inserted between a and b.

Here $a, A_1, A_2, \ldots\ldots\ldots\ldots\ldots\ldots, A_{2n}$, b are in A.P. then given condition

$A_1 + A_2 + \dots\dots\dots + A_{2n} = 2n + 1$
or $(a + A_1 + A_2 + \dots\dots\dots + A_{2n} + b) - (a + b) = 2n + 1$
or $\frac{(2n+2)}{2}(a+b)-(a+b)=2n+1$
or $n(a + b) = 2n + 1$
or $13n = 12n + 6$
or $n = 6$
Hence number of means are 12

41. We can write the sum upto (2n + 1) terms as
$[a + (a + d)](-d) + [(a + 2d) + (a + 3d)](-d) + \dots\dots\dots [(a +(2n - 2) d) + (a + (2n - 1 d]$
$(-d) + (a + 2nd)^2$
$= (-d)[a + (a + d) + (a + 2d) + \dots\dots\dots + a + (2n - 1) d] + (a + 2nd)^2$
$=(-d)\frac{2n}{2}\{a+a+(2n-1)d\}+(a+2nd)^2$
$= -2nad - n(2n - 1)d^2 + a^2 + 4n(ad) + 4n^2d^2$
$= a^2 + 2nad + n(2n + 1)d^2$

42. $T_n=\frac{1}{(n+1)(n+2)(n+3)......(n+k)}$

$v_{n-1}=\frac{1}{(n+1)(n+2)(n+3).....(n+k-2)(n+k-1)}$

$v_n-v_{n-1}=\frac{1}{(n+2)(n+3)....(n+k-1)}\left[\frac{1}{n+k}-\frac{1}{n+1}\right]$

$=\frac{1-k}{(n+1)(n+2)(n+3)......(n+k-1)(n+k)}$

$v_n-v_{n-1}=(1-k)T_n$

$\therefore (1-k)T_n=v_n-v_{n-1}$

$(1-k)T_1=v_1-v_0$

$(1-k)T_2=v_2-v_1$

$(1-k)T_3=v_3-v_2$

....

$(1-k)T_n=v_n-v_{n-1}$

Adding $(1 - k)S_n = v_n - v_0$

or $(1-k)s_n=\frac{1}{(n+1)(n+2).......(n+k)}-\frac{1}{1.2.3......k}$ $(1-k)\lim_{n\to\infty}S_n=0-\frac{1}{\underline{|k}}$

$\lim_{n\to\infty}S_n=\frac{1}{(k-1)\underline{|k}}$

43. Let n = 2m, then
$S_{2m} = 1^2 + 2 \times 2^2 + 3^2 + 2 \times 4^2 + \dots\dots\dots\dots + (2m - 1)^2 + 2(2m)^2$

= 2m (2m + 1)^2/2 = m (2m + 1)
When n = 2m – 1
$1^2 + 2 \times 2^2 + 3^2 + 2 \times 4^2 + \dots\dots\dots + (2m-1)^2$
$= S_{2m} - 2(2m)^2 = m(2m+1)^2 - 2(2m)^2$
$= m[4m^2 + 4m + 1 - 8m] = m(2m-1)^2$
$= n^2(n+1)/2$

44. We have $t_r = \sum_{k=1}^{r} t_k - \sum_{k=1}^{r-1} t_k$

$$= \frac{1}{12} r(r+1)(r+2) - \frac{1}{12}(r-1)(r)(r+1)$$

$$= \frac{1}{4} r(r+1)$$

Now, $\frac{1}{t_r} = \frac{4}{r(r+1)} = 4\left(\frac{1}{r} - \frac{1}{r+1}\right)$

$$\Rightarrow \sum_{r=1}^{n} \frac{1}{t_r} = 4\sum_{r=1}^{n}\left(\frac{1}{r} - \frac{1}{r+1}\right) = 4\left(1 - \frac{1}{n+1}\right) = \frac{4n}{n+1}$$

45. We have $t_r = \frac{(r-1)!}{(r+4)!}$ and $t_{r+1} = \frac{r!}{(r+5)!}$

Now, $rt_r - (r+5)t_{r+1} = \frac{r!}{(r+4)!} - \frac{r!}{(r+4)!} = 0$

$$\Rightarrow rt_r - (r+1)t_{r+1} = 4t_{r+1} \Rightarrow 4\sum_{r=1}^{n-1} t_{r+1} = \sum_{r=1}^{n-1}\left[rt_r - (r+1)t_{r+1}\right]$$

$$\Rightarrow 4(t_2 + t_3 + \dots\dots + t_n) = 1t_1 - nt_n$$

$$\Rightarrow 4(t_1 + t_2 + \dots\dots\dots t_n) = 5t_1 - nt_n$$

$$= 5\left(\frac{0!}{5!}\right) - \frac{n(n-1)!}{(n+4)}$$

$$= \frac{1}{4!} - \frac{n!}{(n+4)!}$$

$$\Rightarrow t_1 + t_2 + \dots\dots\dots t_n = \frac{1}{4}\left[\frac{1}{4!} - \frac{n!}{(n+4)!}\right]$$

46. We have $t_r = \frac{2r-1}{r(r+1)(r+2)} = \frac{2}{(r+1)(r+2)} - \frac{1}{r(r+1)(r+2)}$

$$=2\left(\frac{1}{r+1} - \frac{1}{r+2}\right)\frac{-1}{2}\left(\frac{1}{r(r+1)} - \frac{1}{(r+1)(r+2)}\right)$$

Solving by using v_n – method

we get sum = $\dfrac{n(3n+1)}{4(n+1)(n+2)}$

47. Let t_r denote the rth term of the series, then

$$t_r = \frac{1^3+2^3+........+r^3}{1+3+........+(2r-1)}$$

$$= \frac{\frac{1}{4}r^2(r+1)^2}{r^2} = \frac{1}{4}(r+1)^2$$

$$\Rightarrow \sum_{r=1}^{n} t_r = \frac{1}{4}\sum_{r=1}^{n}(r+1)^2 = \frac{1}{4}\left[\sum_{r=1}^{n+1} r^2 - 1\right]$$

$$= \frac{1}{4}\left[\frac{(n+1)(n+2)(2n+3)}{6} - 1\right]$$

$$= \frac{1}{24}\left[2n^3+9n^2+13n+6-6\right]$$

$$= \frac{n}{24}\left(2n^2+9n+13\right)$$

48. $27pqr \ge (p+q+r)^3$

$$\Rightarrow (pqr)^{1/3} \ge \left(\frac{p+q+r}{3}\right)$$

$\Rightarrow p = q = r$

Also 3p + 4q + 5r = 12 $\Rightarrow p = q = r = 1$

49. As odd number of AM, G.M and H.M. are inserted between a & b.
So, middle term of AP is AM = a_n
middle term of GP is GM = b_n
middle term of HP is HM = c_n
$\therefore$ a_n, b_n, c_n are in G.P.
$\therefore$ D = discriminant of quadratic equation < 0
$\therefore$ roots are imagnary

50. $$\frac{\tan^n A + \tan^n B + \tan^n C}{3} \ge \left(\frac{\tan A + \tan B + \tan C}{3}\right)^n$$

(Arithmetic mean of m^{th} power of numbers)

$$\frac{\tan^n A + \tan^n B + \tan^n C}{3} \ge \left(\frac{\tan A + \tan B + \tan C}{3}\right)^n$$

$\left(\text{since } \tan A + \tan B + \tan c \ge 3\sqrt{3}\right)$ $\Rightarrow \tan^n A + \tan^n B + \tan^n C \ge 3^{\frac{n}{2}+1}$

LEVEL - III

51. $$\frac{A+B}{AB} = 2\frac{A+B}{2AB} = \frac{2}{H}$$

$$\therefore E=\frac{2}{H}(1+1+1+...+1)=\frac{2n}{H}$$

52. p = Infinite G.P.

where a = 1, r = – $\tan^2 x$

$$\therefore p=\frac{a}{1-r}=\frac{1}{1+\tan^2 x}=\cos^2 x,\quad q=\frac{1}{1+\cot^2 y}=\sin^2 y$$

$$\therefore S=\frac{1}{1-\tan^2 x\cot^2 y}=\frac{1}{1-\left(\frac{1-\cos^2 x}{\cos^2 x}\right)\left(\frac{1-\sin^2 y}{\sin^2 y}\right)}$$

$$S=\frac{pq}{p+q+1}=\frac{1}{\left\{\frac{1}{p}+\frac{1}{q}-\frac{1}{pq}\right\}}$$

53. Since $S_n=\sum_{r=1}^{n}\frac{8r}{4r^4+1}$

$$=\sum_{r=1}^{n}\frac{8r}{(2r^2-2r+1)(2r^2+2r+1)}$$

$$=2\sum_{r=1}^{n}\frac{(2r^2+2r+1)-(2r^2-2r+1)}{(2r^2-2r+1)(2r^2+2r+1)}=2\sum_{r=1}^{n}\left(\frac{1}{2r^2-2r+1}-\frac{1}{2r^2+2r+1}\right)$$

$$=2\left\{\frac{1}{1}-\frac{1}{5}+\frac{1}{5}-\frac{1}{13}+.......+\frac{1}{2n^2-2n+1}-\frac{1}{2n^2+2n+1}\right\}$$

$$=2\left\{1-\frac{1}{2n^2+2n+1}\right\}$$

$$=\frac{2(2n^2+2n)}{(2n^2+2n+1)}=\frac{4n^2+4n}{(2n^2+2n+1)}$$

$$\therefore S_{16}=\frac{4(16)^2+4(16)}{2(16)^2+2(16)+1}=\frac{1088}{545}$$

54. Since $\sum_{r=1}^{n}(2r+1)f(r)=\sum_{r=1}^{n}(r^2+2r+1-r^2)f(r)$

$$=\sum_{r=1}^{n}\left\{(r+1)^2-r^2\right\}f(r)$$

$$=\sum_{r=1}^{n}\left\{(r+1)^2 f(r)-(r+1)^2 f(r+1)+(r+1)^2 f(r+1)-r^2 f(r)\right\}$$

$$=\sum_{r=1}^{n}(r+1)^2\left\{f(r)-f(r+1)\right\}+\sum_{r=1}^{n}\left\{(r+1)^2 f(r+1)-r^2 f(r)\right\}$$

$$= -\sum_{r=1}^{n}\frac{(r+1)^2}{(r+1)} + \sum_{r=1}^{n-1}(r+1)^2 f(r+1) + (n+1)^2 f(n+1) - \sum_{r=1}^{n} r^2 f(r)$$

$$= -\sum_{r=1}^{n}(r+1) + \left\{2^2 f(2) + 3^2 f(3) + ... + n^2 f(n)\right\}$$

$$+ (n+1)^2 f(n+1) - \left\{1^2 f(1) + 2^2 f(2) + 3^2 f(3) + ... + n^2 f(n)\right\}$$

$$= -\sum_{r=1}^{n} r - \sum_{r=1}^{n} 1 + (n+1)^2 f(n+1) - 1^2 f(1) \qquad = -\frac{n(n+1)}{2} - n + (n+1)^2 f(n+1) - f(1)$$

$$= (n+1)^2 f(n+1) - \frac{n(n+3)}{2} - 1 \qquad = (n+1)^2 f(n+1) - \frac{(n^2+3n+2)}{2}$$

55. $\because \log_y x, \log_z y, \log_x z$ are in G.P.

$\therefore (\log_z y)^2 = \log_y x \log_x z$

or $(\log_z y)^2 = \dfrac{1}{\log_z y}$

or $(\log_z y)^3 = 1 \Rightarrow \log_z y = 1$

it is possible when y = z (x, y, z > 0)

from $2x^4 = y^4 + z^4$

$2x^4 = 2y^4$

x = y = z

from xyz = 8,

$x^3 = 8$

$\therefore x = 2 \Rightarrow x = y = z = 2$

56. Let r be the common ratio then b = ar, c = ar^2 and $\log_c a$, $\log_b c$, $\log_a b$ are in A.P.

$\dfrac{\log_e a}{\log_e c}, \dfrac{\log_e c}{\log_e b}, \dfrac{\log_e b}{\log_e a}$ are in A.P.

$\Rightarrow \dfrac{\log_e a}{\log_e (ar^2)}, \dfrac{\log_e (ar^2)}{\log_e ar}, \dfrac{\log_e ar}{\log_e a}$ are in A.P.

so $\dfrac{\log_e a}{\log_e a + 2\log_e r}, \dfrac{\log_e a + 2\log_e r}{\log_e a + \log_e r}, \dfrac{\log_e a + \log_e r}{\log_e a}$ are in A.P.

Putting $\dfrac{\log_e r}{\log_e a} = x$

we get

$\dfrac{1}{1+2x}, \dfrac{1+2x}{1+x}, 1+x$ are in A.P.

$\therefore \dfrac{2(1+2x)}{(1+x)} = (1+x) + \dfrac{1}{(1+2x)}$

$\Rightarrow 2x^3 - 3x^2 - 3x = 0$

since a, b, c are distinct so $r \neq 1$, so $x \neq 0$

$\therefore 2x^2 - 3x - 3 = 0$

$x = \frac{1}{4}\left(3 \pm \sqrt{33}\right)$

then $\left(1+x\right) - \frac{1}{\left(1+2x\right)} = 3$, so the common differences of A.P. is 3/2.

57. Since,

$$t_n = \frac{n^5 + n^3}{n^4 + n^2 + 1} = n - \frac{n}{n^4 + n^2 + 1}$$

$$= n + \frac{1}{2\left(n^2 + n + 1\right)} - \frac{1}{2\left(n^2 - n + 1\right)}$$

$\therefore$ sum of n terms $S_n = \sum_{n=1}^{n} t_n$

$$S_n = \sum_{n=1}^{n} n + \frac{1}{2}\left\{\left(\frac{1}{n^2 + n + 1} - \frac{1}{n^2 - n + 1}\right)\right\}$$

$$= \left(1 + 2 + 3 + ... + n\right) + \frac{1}{2}\left\{\frac{1}{3} - 1 + \frac{1}{7} - \frac{1}{3} + \frac{1}{13} - \frac{1}{7} + ... + \frac{1}{n^2 + n + 1} - \frac{1}{n^2 - n + 1}\right\}$$

$$= \frac{n\left(n+1\right)}{2} + \frac{1}{2}\left\{-1 + \frac{1}{n^2 + n + 1}\right\}$$

$$= \frac{n^2}{2} + \frac{n}{2} - \frac{1}{2} + \frac{1}{2} + . \frac{1}{\left(n^2 + n + 1\right)}$$

$$= \left(\frac{n}{\sqrt{2}} + \frac{1}{2\sqrt{2}}\right)^2 - \frac{1}{8} - \frac{1}{2} + \frac{1}{2n^2 + 2n + 1}$$

$$= \left(\frac{n}{\sqrt{2}} + \frac{1}{2\sqrt{2}}\right)^2 - \frac{5}{8} + \frac{1}{\left(n\sqrt{2} + \frac{1}{\sqrt{2}}\right)^2 + \frac{3}{2}}$$

but given $S_n = a_n^2 + a + \frac{1}{b_n^2 + b}$

On comparing we get

$$a_n = \left(\frac{n}{\sqrt{2}} + \frac{1}{2\sqrt{2}}\right), a = -\frac{5}{8}, b_n = \left(n\sqrt{2} + \frac{1}{\sqrt{2}}\right), b = \frac{3}{2}$$

Hence, $\frac{a_n}{b_n} = \frac{1}{2}$, which is constant.

58. $T_1 = \frac{1}{\sqrt{a_1} + \sqrt{a_2}} = \frac{\sqrt{a_1} - \sqrt{a_2}}{-d}$

$\therefore S_n = \Sigma T_1 = -\frac{1}{d}\left[\sqrt{a_1} - \sqrt{a_n}\right]$

$\therefore$ The terms will cancel diagonally

$$\therefore S_n = -\frac{1}{d}\frac{a_1 - a_n}{\sqrt{a_1}+\sqrt{a_n}}$$

Now put $a_n = a_1 + (n-1)d$

$$= -\frac{1}{d}\left[\frac{-(n-1)d}{\sqrt{a_1}+\sqrt{a_n}}\right] = \frac{n-1}{\sqrt{a_1}+\sqrt{a_n}}$$

59. We have
1111 + ... 1 (91 times)
= 1 + 10 + 10^2 + ... + 10^{90}

$$= \frac{10^{91}-1}{10-1}$$

$$= \frac{(10^{91}-1)}{(10^7-1)}.\frac{(10^7)-1}{(10-1)} = \frac{(10^7)^{13}-1}{10^7-1}.\frac{10^7-1}{(10-1)}$$

= (10^{84} + 10^{77} + 10^{70} + ... 10^7 + 1) (10^6 + 10^5 + 10^4 + 10^3 + 10^2 + 10 + 1)
= product of two integers (> 2). so is not a prime number.

60. Let the three digit be a, ar, ar^2 then according to hypotesis
100a + 10 ar + ar^2 + 792 = 100 ar^2 + 10 ar + a , i.e. a (r^2 – 1) = 8 and a, ar + 2, ar^2 are in A.P. then 2 (ar + 2) = a + ar^2 so r = 3 & a = 1. Thus digits are 1, 3, 9 and so the required number is 931.

ONE OR MORE THAN ONE CORRECT ANSWERS

LEVEL-I

1. Let the numbers in A.P. are a, (a + 2), (a + 4)....... a + (a + 2) + (a + 4) + ... = 155

$$\frac{n}{2}\left[2a+(n-1)2\right]=155 \qquad n\left[a+(n-1)\right]=155$$

n can not be even as sum is an odd number

2. (A), (B) on rationalizing each term, we get series

$$= \frac{\sqrt{7}-\sqrt{3}}{7-3}+\frac{\sqrt{11}-\sqrt{7}}{11-7}+\frac{\sqrt{15}-\sqrt{11}}{15-11}+\text{.........upto n term}$$

$= \frac{1}{4}\left[\sqrt{3+4n}-\sqrt{3}\right]$ which is equal to $\frac{n}{\sqrt{3+4n}+\sqrt{3}}$

choices (A) and (B) are correct

(C) Since $\frac{n}{\sqrt{3+4n}+\sqrt{3}} < n$, choice c is also correct

(D) And $\frac{n}{\sqrt{3+4n}+\sqrt{3}} > \frac{n}{\sqrt{4n}} > \frac{\sqrt{n}}{2}$

3. We have

$b_3 > 4b_2 - 3b_1 \Rightarrow b_1r^2 > 4b_1r - 3b_1$

$\Rightarrow r^2 > 4r - 3 \qquad [\because b_1 > 0]$

$\Rightarrow r^2 - 4r - 3 > 0 \Rightarrow (r-3)(r-1) > 0$

$\Rightarrow r > 3$ or $r < 1$

Since r = 3.5 and r = 5.2 are both greater than 3, (c) and (d) are also true.

4. We have 1072 < 10 (2a + 19d) < 1162 and a + 5d = 32

$\Rightarrow$ 1072 < 640 + 90d < 1162

$\therefore \dfrac{432}{90} < d < \dfrac{522}{90}$ and d is natural number, so d = 5 $\Rightarrow$ a = 7

5. Let the first four terms be $\dfrac{a}{r^3}, \dfrac{a}{r}, a, ar^3$ then $\left(\dfrac{a}{r^3}\right)\left(\dfrac{a}{r}\right)(ar)(ar^3) = 4$

$\Rightarrow a^4 = 4$

$\Rightarrow a = \pm\sqrt{2}$

Also given $\dfrac{a}{r} = \dfrac{1}{ar^3} \Rightarrow r^2 = \dfrac{1}{2}$ $\quad r = \pm\dfrac{1}{\sqrt{2}}$

so, the sum to infinite series $S = \dfrac{\frac{a}{r^3}}{1-r^2}$ ($\because$ first term = $\dfrac{a}{r^3}$ and common ratio = r^2)

6. $\dfrac{\sin x}{6}. \cos x, \tan x$ are in G.P.

$\Rightarrow \cos^2 x = \dfrac{\sin x . \tan x}{6}$

$\Rightarrow 6\cos^3 x + \cos^2 x - 1 = 0$

Put cos x = t $\Rightarrow 6t^3 + t^2 - 1 = 0$

$\Rightarrow (2t-1)(3t^2 + 2t + 1) = 0$

As the quadratic factor has imaginary roots.

$\therefore$ t = 1/2 i.e.., cos x = 1/2

$\Rightarrow x = \pm\dfrac{\pi}{3} + 2k\pi$

7. $\dfrac{x^2+2}{x} = \dfrac{x^3+10}{x^2+2} \Rightarrow x = 2, \dfrac{1}{2}$

Given G.P. becomes

2, 6, 18, 45,... or $\dfrac{1}{2}, \dfrac{9}{4}, \dfrac{81}{8}, \dfrac{729}{16}$.....

$\Rightarrow$ Next term is $\frac{729}{16}$ or 54.

8. $S_n = \frac{n}{2}\left[2a' + (n-1)d\right] = a + bn + cn^2$

$\Rightarrow na' + \frac{n(n-1)d}{2} = a + bn + cn^2$

$\Rightarrow \left(a' - \frac{d}{2}\right)n + \frac{n^2 d}{2} = a + bn + cn^2$

On comparing coefficient, we get

$a = 0, b = a' - \frac{d}{2}, c = \frac{d}{2}$

9. Take a = 1, b = 2. c = 3 (As a, b, c are in A.P.)

We get, 2^{bx+1} is G.M. of 2^{ax+1} and 2^{cx+1} for x $\neq$ 0.

10. $a_1a_2 = \frac{a_1a_2d}{d} = \frac{a_1a_2}{d}\left(\frac{1}{a_2} - \frac{1}{a_1}\right) = \frac{a_1 - a_2}{d}$

Similarly $a_2a_3 = \frac{a_2 - a_3}{d}, a_3a_4 = \frac{a_3 - a_4}{d}$ and so on

$a_1a_2 + a_2a_3 + a_3a_4 + ... + a_{n-1}a_n$

$= \frac{1}{d}\left[a_1 - a_2 + a_3 - a_4 + ... + a_{n-1} - a_n\right]$

$= \frac{a_1 - a_n}{d}$ ...... (i)

But $\frac{1}{a_n} = \frac{1}{a_1} + (n-1)d \Rightarrow \frac{\frac{1}{a_n} - \frac{1}{a_1}}{n-1} = d$

Equation (i) becomes (n – 1) a_1a_n

11. On solving $\frac{\frac{2ab}{a+b}}{\sqrt{ab}} = \frac{4}{5}$ as a quadratic in $\frac{a}{b}$,

we get $\frac{a}{b}$ = 4, 1/4

a and b are the correct choices.

12. Let A, a_1, a_2, B be in A. P.

$\therefore a_1 = A + \frac{B-A}{3} = \frac{2A+B}{3}$

$\therefore a_2 = A + 2.\frac{B-A}{3} = \frac{A+2B}{3}$

Also A, g_1, g_2, B are in G.P.

$\therefore \frac{B}{A} = r^3$

$\therefore g_1 = Ar = A\left(B/A\right)^{1/3}$

$\therefore g_2 = Ar^2 = A\left(B/A\right)^{2/3}$

$\therefore g_1 g_2 = A^2\left(B/A\right) = AB$

Also A, h_1, h_2, B are in H.P.

$\therefore \frac{1}{A}, \frac{1}{h_1}, \frac{1}{h_2}, \frac{1}{B}$ are in AP

$\therefore \frac{1}{h_1} = \frac{1}{A} + \frac{1}{3}\left(\frac{1}{B} - \frac{1}{A}\right) = \frac{1}{A} + \frac{A-B}{3AB}$

$\Rightarrow \frac{1}{h_1} = \frac{3B+A-B}{3AB} = \frac{A+2B}{3AB} \Rightarrow h_1 = \frac{3AB}{A+2B}$ and

$\frac{1}{h_2} = \frac{1}{A} + \frac{2}{3}\left(\frac{1}{B} - \frac{1}{A}\right) = \frac{3B+2\left(A-B\right)}{3AB}$

$\Rightarrow \frac{1}{h_2} = \frac{2A+B}{3AB}$

$\therefore h_2 = \frac{3AB}{2A+B}$

Obviously $g_1 g_2 = AB = a_1 h_2 = a_2 h_1$

13. $T_r = \frac{1+2+3+.........+r}{1^3+2^3+...........+r^3}$

$\frac{2}{r\left(r+1\right)} = 2\left(\frac{1}{r} - \frac{1}{\left(r+1\right)}\right)$

$\Rightarrow S_n = \sum_{r=1}^{n} T_r = 2\sum_{r=1}^{n}\left(\frac{1}{r} - \frac{1}{r+1}\right) = 2\left(1 - \frac{1}{n+1}\right)$

which can not be greater than 2

14. $2q = p + r$, $q^2 - 4pr \geq 0$

Eliminate q to obtain $p^2 + r^2 - 14\,pr \geq 0$

Which gives (a) and (c)

Eliminate r to obtain

$q^2 - 8pq + 4p^2 \geq 0$
which gives (b) and (d)

15. $A_1 = a + \frac{1}{3}(b-a), A_2 = b + \frac{2}{3}(b-a)$

$\Rightarrow A_1 + A_2 = a + b$

Similarly, $G_1 = a\left(\frac{b}{a}\right)^{1/3}, G_2 = a\left(\frac{b}{a}\right)^{2/3}$

$\Rightarrow G_1G_2 = ab$

and $\frac{1}{H_1} = \frac{1}{a} + \frac{1}{3}\left(\frac{1}{b} - \frac{1}{a}\right), H_2 = \frac{1}{a} + \frac{2}{3}\left(\frac{1}{b} - \frac{1}{a}\right)$

Now, $\frac{1}{H_1} + \frac{1}{H_2} = \frac{1}{a} + \frac{1}{b}$

$\Rightarrow \frac{H_1 + H_2}{H_1H_2} = \frac{a+b}{ab} = \frac{A_1 + A_2}{G_1G_2}$

$\Rightarrow \frac{G_1G_2}{H_1H_2} = \frac{A_1 + A_2}{H_1 + H_2}$

Now, $H_1 + H_2 = \frac{3ab}{a+2b} + \frac{3ab}{2a+b} \quad = \frac{9ab(a+b)}{(a+2b)(2a+b)}$

$\Rightarrow \frac{A_1 + A_2}{H_1 + H_2} = \frac{2(a^2+b^2)+5ab}{9ab}$ Thus, $\frac{G_1G_2}{A_1A_2} - \frac{5}{9} = \frac{2}{9}\left(\frac{a}{b} + \frac{b}{a}\right)$

16. The given series can be written as

$100\left[\frac{2-1}{1.2} + \frac{3-2}{2.3} + ... + \frac{100-99}{99.100}\right]$

$= 100\left[\left(\frac{1}{1} - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + ... + \left(\frac{1}{99} - \frac{1}{100}\right)\right]$

= 99

17. $5S_n = (1)(5)^2 + (2)(5^3) + ... + (n-1)5^n + (n)5^{n+1}$
Subtracting from S_n, we obtain

$-4S_n = 5 + 5^2 + ... + 5^n - n(5^{n+1}) = \frac{5(5^n - 1)}{4} - n(5^{n+1})$

$\therefore S_n = \frac{1}{16}\left[(4n-1)5^{n+1} + 5\right]$

18. Let the four numbers be a, a + d, a + 2d, $(a + d).r^2$.
where d is the common difference of A.P. and r is common ratio of the G.P.

a = 6 and r = $\frac{1}{2}$ is given

a + d, a + 2d, (a + d) r^2 are in G.P.

$\Rightarrow$ $(a + 2d)^2 = (a + d)^2 r^2$ $\Rightarrow (6+2d)^2 = (6+d)^2 . \frac{1}{4}$

$\Rightarrow 4(6+2d)^2 = (6+d)^2$ $\Rightarrow 6+2d = (6+d).\frac{1}{2}$

$\Rightarrow$ d = –2 $\Rightarrow$ The 4 numbers are 6, 4, 2, 1.

19. Let a_n denotes the side of the square S_n then

$a_n = \sqrt{2} a_{n+1}$

$\Rightarrow \frac{a_{n+1}}{a_n} = \frac{1}{\sqrt{2}}$

$\Rightarrow a_n = a_1 \left(\frac{1}{\sqrt{2}}\right)^{n-1}$ (G.P. formula) = $10\left(\frac{1}{\sqrt{2}}\right)^{n-1}$

Now, we must have $a_n^2 < 1$

$\Rightarrow 100\left(\frac{1}{\sqrt{2}}\right)^{2n-2} < 1$

$\Rightarrow 2^n > 200$ $\Rightarrow n > 8$

20. Let the positive integers in A.P. are a, a + d, a + 2d,....
according to question a + a + d + a + 2d = 12
$\Rightarrow$ a + d = 4 .... (i)
also a + 3d + a + 5d = 14
$\Rightarrow$ a + 4d = 7 .... (ii)
From (i) and (ii) a = 3, d = 1
Hence x_5 = a + 4d = 7

LEVEL - II

21. As a, b, c are in H.P., 1/a, 1/b, 1/c are in A.P.
Let the common difference of this A.P. be d Now,

$E = \left(\frac{1}{c}+d\right)\left(\frac{1}{c}-d\right) = \frac{1}{c^2} - d^2 = \frac{1}{c^2} - \left(\frac{1}{c}-\frac{1}{b}\right)^2 = \frac{2}{bc} - \frac{1}{b^2}$

Next, $E = \frac{1}{c^2} - \frac{1}{4}\left(\frac{1}{c}-\frac{1}{a}\right)^2 = \frac{1}{4}\left(\frac{3}{c^2}+\frac{2}{ca}-\frac{1}{a^2}\right)$

and $E = \left(\frac{2}{b}-\frac{1}{a}\right)^2 - \left(\frac{1}{b}-\frac{1}{a}\right)^2 = \frac{3}{b^2} - \frac{2}{ab}$

22. Take $a = \frac{1}{p-3q}, b = \frac{1}{p-q}, c = \frac{1}{p+q}, d = \frac{1}{p+3q}$

Then a + d > b + c easily follows

Since (a + d) – (b + c)

$$= \frac{2p}{p^2 - 9q^2} - \frac{2p}{p^2 - q^2} = 2p\left[\frac{8q^2}{(p^2 - 9q^2)(p^2 - q^2)}\right]$$

Which is positive $(\because a, b, c, d > 0)$

Also $ad - bc = \frac{1}{p^2 - 9q^2} - \frac{1}{p^2 - q^2}$

$$= \frac{8q^2}{(p^2 - 9q^2)(p^2 - q^2)} > 0$$

23. If n is odd then n – 1 is even and $S_n = S_{n-1} + n^2$

$$= \frac{(n-1)n^2}{2} + n^2 = \frac{n^2}{2}(n+1)$$

since $\frac{n^2}{2}(n+1)$ is even if n is of the from 4 *l* + 3 .

24. Let the sides be a, ar, ar_2. If r > 1, then
$(ar^2)^2 = (a)^2 + (ar)^2$ (since in this case $(ar)^2$ will be the hypotaneous i.e., the largest side)

$$\Rightarrow r^4 = 1 + r^2 \Rightarrow r^2 = \frac{1+\sqrt{5}}{2}, \left(r^2 = \frac{1-\sqrt{5}}{2}\right) \text{ is not possible}$$

$$\Rightarrow r = \sqrt{\frac{1+\sqrt{5}}{2}}$$

If 0 < r < 1 then a is the largest side

$$\therefore a^2 = (ar)^2 + (ar^2)^2$$

$$\therefore r = \sqrt{\frac{\sqrt{5}-1}{2}}$$

25. we have 2b = a + c and $b^2 = \frac{2a^2c^2}{a^2 + c^2}$ ....(i)

On eliminating b, we get

$$8a^2c^2 = (a^2 + c^2 + 2ac)(a^2 + c^2)$$

which can be arranged as

$$(a^2 + c^2 - 2ac)(a^2 + c^2 + 4ac) = 0$$

$\Rightarrow$ either a = c or $(a + c)^2 + 2ac = 0$
If a = c then a = b = c
$\Rightarrow$ a, b, c may be treated as three numbers in G.P.
If $(a + c)^2 + 2ac = 0$, then by using (i) choice (d) follows.

26. Given $(a + nd)^2 = (a + md)(a + rd)$

$$\Rightarrow \left(\frac{a}{d} + n\right)^2 = \left(\frac{a}{d} + m\right)\left(\frac{a}{d} + r\right) \quad \text{... (i)}$$

Also n = $\frac{2mr}{m+r} \Rightarrow mr = \frac{(m+r)n}{2}$ ... (ii)

Now from (i), $\left(\frac{a}{d}\right)^2 + 2\left(\frac{an}{d}\right) + n^2 = \left(\frac{a}{d}\right)^2 + (m+r)\frac{a}{d} + mr$

$\Rightarrow \frac{a}{d} = \frac{n^2 - mr}{m+r-2n} = \frac{n^2 - \frac{(m+r)n}{2}}{m+r-2n}$ from (ii)

$\therefore \frac{a}{d} = -\frac{n}{2} = -\frac{mr}{m+r}$

27. Let roots be $\alpha - \beta$, α, $\alpha + \beta$, so that

$\mathbf{3\alpha = -b \Rightarrow \alpha = -b/3}$

$\therefore -\frac{\mathbf{b^3}}{\mathbf{27}} + \frac{\mathbf{b^3}}{\mathbf{9}} - \frac{\mathbf{bc}}{\mathbf{3}} + \mathbf{d} = \mathbf{0}$

$\Rightarrow \mathbf{2b^3 - 9bc + 27d = 0}$

Next, roots be α/β, α, $\alpha\beta$, so that

$\alpha^3 = -d$ or $\alpha = (-d)^{1/3}$

$\therefore -d + b(-d)^{2/3} + c(-d)^{1/3} + d = 0$

$\Rightarrow \mathbf{b^3 d = c^3}$

28. As all x_i's are positive integers and $x_1.x_2.x_3.x_4 = 64$

$\Rightarrow x_1, x_2, x_3, x_4$ must be power of 2.

Then, amongst all possible values of $x_1.x_2.x_3.x_4 = 64$ and in that case $x_5 = 16$.

29. a_n is sum of reciprocals of natural numbers starting at n + 1 and ending at 3n

$\therefore a_2 = \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} = \frac{19}{20}$

$\therefore a_{n+1} = \frac{1}{n+2} + \frac{1}{n+3} + ... + \frac{1}{3n} + \frac{1}{3n+1} + \frac{1}{3n+2} + \frac{1}{3n+3}$

$\Rightarrow a_{n+1} - a_n = \frac{1}{3n+1} + \frac{1}{3n+2} + \frac{1}{3n+3} - \frac{1}{n+1}$

$= \frac{1}{3n+1} + \frac{1}{3n+2} - \frac{2}{2n+3}$

$= \frac{1}{3n+1} + \frac{1}{3n+2} - \frac{2}{3n+3}$

$= \frac{9n+5}{(3n+1)(3n+2)(3n+3)}$

30. $an^4 + bn^3 + an^2 + dn + e$

$= \mathbf{2\sum_{r=1}^{n} r(r+1)(r+2) - \sum_{r=1}^{n} r(r+1)}$

$$=\frac{2}{4}n(n+1)(n+2)(n+3)-\frac{1}{3}n(n+1)(n+2)$$

$$=\frac{1}{6}\left(3n^4+16n^3+27n^2+14n\right)$$

LEVEL - III

31. $$\frac{\frac{p}{2}\left[a_1+(p-1)d\right]}{\frac{q}{2}\left[a_1+(q-1)d\right]}=\frac{p^2}{q^2}\Rightarrow\frac{a_1+(p-1)d}{a_1+(q-1)d}=\frac{p}{q}$$

For $\frac{a_6}{a_{21}}$, put $p=6, q=21$

$$\frac{a_6}{a_{21}}=\frac{2}{7}<1$$

32. The relation $2b=\frac{a+b}{1-ab}+\frac{b+c}{1-bc}$ will yield a + c = 2 abc

33. $\frac{1}{a},\frac{1}{b},\frac{1}{c}$ are in A.P.

$\Rightarrow\frac{a+b+c}{a},\frac{a+b+c}{b},\frac{a+b+c}{c}$ are in A.P.

$\Rightarrow\frac{b+c}{a},\frac{c+a}{b},\frac{a+b}{c}$ are in A.P. (Subtracting 1 from each term)

$\Rightarrow\frac{b+c}{a}-1,\frac{c+a}{b}-1,\frac{a+b}{c}-1$ are in A.P. (Subtracting 1 from each term)

$\Rightarrow\frac{b+c-a}{a},\frac{c+a-b}{b},\frac{a+b-c}{c}$ are in A.P.

Also $b=\frac{2ac}{a+c}$ so

$$\frac{1}{b-a}+\frac{1}{b-c}=\frac{2b-(a+c)}{(b-a)(b-c)}=\frac{2b-(a+c)}{b^2-b(a+c)+ac}$$

$$=\frac{2b-2ac/b}{b^2-b(2ac)b+ac}=\frac{2}{b}.\frac{b^2-ac}{b^2-ac}=\frac{2}{b}$$

34. Let 1, 5, 25 be the p^{th}, q^{th}, r^{th} terms of an A.P. with common difference d, then (q – p) d = 5 – 1 and (r – p) d = 25 – 1

$$\Rightarrow\frac{q-p}{1}=\frac{r-p}{6}=k(say)$$

$\Rightarrow q=p+k, r=p+6k$ where k is any natural number.

Let 1, 5, 25 be the p^{th}, q^{th}, r^{th} terms of a G.P., with common ratio R, then $R^{q-p}=5$, $R^{r-p}=25$

$\therefore r-p=2q-2p \Rightarrow r+p-2q=0$

There exist infinitely many triplets of natural numbers satisfying this relation.

35. $S_2=\frac{1}{1^2}+\frac{1}{2^2}=\frac{5}{4}, T_2=\frac{3}{2}$

$\Rightarrow S_2<\frac{3}{2}$

(a) is true

If $S_k < T_k$

Then $\frac{1}{1^2}+\frac{1}{2^2}+...........+\frac{1}{k^2}<2-\frac{1}{k}$

on adding $\frac{1}{(1+k)^2}$ on both side $\frac{1}{1^2}+\frac{1}{2^2}+..........+\frac{1}{k^2}+\frac{1}{(1+k)^2}<2-\frac{1}{k}+\frac{1}{(k+1)^2}$

Now $2-\frac{1}{k}-\frac{1}{(k+1)^2}<2-\frac{1}{k+1}$ will be true if $\frac{1}{k}+\frac{1}{(k+1)^2}>\frac{1}{k+1}$

or (k + 1)2 + k $\geq$ k (k + 1)

or $k^2 + 3k + 1 \geq k^2 + k$ which is true.

$\Rightarrow S_{k+1} < T_{k+1}$

LINKED COMPREHENSION TYPE

1. As A.M. $\geq$ G.M.

$$\frac{\frac{b}{c}+\frac{c}{b}+\frac{c}{a}+\frac{a}{c}+\frac{a}{b}+\frac{b}{a}}{6}\geq\left(\frac{c}{b}\times\frac{b}{c}\times\frac{c}{a}\times\frac{a}{c}\times\frac{a}{b}\times\frac{b}{a}\right)^{\frac{1}{6}}$$

or, $\left(\frac{b^2+c^2}{bc}\right)+\frac{c^2+a^2}{ca}+\frac{a^2+b^2}{ab}\geq 6$

or, $a(b^2+c^2)+b(c^2+a^2)+c(a^2+b^2)\geq 6abc$

$\therefore$ minimum value of $a(b^2+c^2)+b(c^2+a^2)+c(a^2+b^2)$ is 6 abc

according to question $\lambda abc = 6abc$

$\therefore \lambda = 6$

2. As a, b, c,d, e, f, are (+) ve

(a+f) (b +e) (c+d) > 0

So, X > 0 ....(1)

As A. M. $\geq$ G.. M.

$$\frac{(a+f)+(b+e)+(c+d)}{3}\geq\left[(a+f)(b+e)(c+d)\right]^{\frac{1}{3}}$$

or, $\sqrt[3]{x}\leq\frac{3}{3}$

or, x $\leq$ 1................(2)

from (1) and (2) 0 < x $\leq$ 1

3. By weightage mean

$$\frac{3\left(\frac{a}{3}\right)+4\left(\frac{b}{4}\right)}{7} \geq 7\sqrt{\left(\frac{a}{3}\right)^3\left(\frac{b}{4}\right)^4}$$

or, $\frac{a+b}{7} \geq 7\sqrt{\frac{a^3}{3^3} \times \frac{b^4}{4^4}}$

Paragraph for Question Nos. 4 to 6

Sol. Let terms of G.P. are $\frac{a}{r}, a, ar$

Given $\frac{a}{r} + a + ar = \alpha s$

or $a\left(\frac{1+r+r^2}{r}\right) = \alpha s$ ..... (i)

and $\frac{a^2}{r^2} + a^2 + a^2r^2 = s^2$

or $a^2\left(\frac{r^4+r^2+1}{r^2}\right) = s^2$ ...... (ii)

on squaring (i) & dividing by (ii)

we get . $\frac{r^2+r+1}{r^2-r+1} = \alpha^2$ .... (iii)

4. by (iii), $r^2\left(1-\alpha^2\right) + r\left(1+\alpha^2\right) + \left(1-\alpha^2\right) = 0$

Since increasing G.P. $\Rightarrow D > 0$

$\frac{1}{3} < \alpha^2 < 3$

5. If $\alpha^2 = 2$ then $r^2 + r + 1 = 2r^2 - 2r + 2$

$\Rightarrow r = \frac{3+\sqrt{5}}{2}$

6. If $\alpha^2 = 3$

$r^2 + r + 1 = 3r^2 - 3r + 3$

$2r^2 - 4r + 2 = 0 \Rightarrow r = 1$

Paragraph for Question Nos. 7 to 9

Sol. $x_1 + x_2 = -b/a, \quad x_1x_2 = c/a,$

$x_3 + x_4 = -q/p, \quad x_3x_4 = r/p,$

7. $x_2 - x_1 = \frac{1}{x_4} - \frac{1}{x_3} \qquad (x_2 - x_1)^2 = \frac{(x_3 - x_4)^2}{(x_4x_3)^2}$

$\Rightarrow \frac{(x_2 - x_1)^2}{(x_4 - x_3)^2} = \frac{1}{(x_3x_4)^2}$

$\Rightarrow \frac{(x_2 + x_1)^2 - 4x_1x_2}{(x_3 + x_4)^2 - 4x_3x_4} = \frac{1}{(x_3x_4)^2}$

On putting values we get

$\frac{b^2 - 4ac}{q^2 - 4pr} = \frac{a^2}{r^2}$

8. $\frac{x_2}{x_1} = \frac{x_4}{x_3} \Rightarrow \frac{x_2 + x_1}{x_1} = \frac{x_4 + x_3}{x_3}$

$\frac{(x_1 + x_2)^2}{(x_3 + x_4)^2} = \frac{x_1^{\,2}}{x_3^2} = \frac{x_1x_2}{x_3x_4}$

on putting value, we get $q^2 = pr$

9. $x_2 = x_1r', x_3 = x_1r'^2, x_4 = x_1r'^3$

$x_1x_2 = x_1^{\,2}r' = c/a$ .... (i)

$x_3x_4 = x_1^{\,2}r'^5 = r/p$ .... (ii)

dividing (ii) by (i) $r'^4 = \frac{ra}{pc} \Rightarrow \left(\frac{ra}{pc}\right)^{1/4} = r'$

Paragraph for Question Nos. 10 to 12

10. **Sol:** $a_1 = 1, a_{n+1}^2 = (2\alpha^2 - 2\alpha + 1)^n$

$\therefore \sum_{n=1}^{\infty} A_n = \frac{1}{2\alpha - 2\alpha^2} = \frac{8}{3}$

$\Rightarrow 2\alpha - 2\alpha^2 = 3/8$

$\Rightarrow 16\alpha^2 - 16\alpha + 3 = 0$

$\Rightarrow \alpha = 1/4,\ 3/4$

11. Diagonal of (n + 1)th square = $\sqrt{2}a_{n+1}$

$\Rightarrow a_n^2 = 2a_{n+1}^2$

$\Rightarrow a_n^2 = 2\left(2\alpha^2 - 2\alpha + 1\right)a_n^2$

$\Rightarrow \alpha = 1/2$

12. $P_1 = 4$

$P_n = 4a_n = 4\left(1 - 2\alpha + 2\alpha^2\right)^{(n-1)/2} \quad = 4\left(\frac{5}{8}\right)^{(n-1)/2}$

$\therefore \sum_{n=1}^{\infty} P_n = \frac{4\sqrt{8}}{\sqrt{8} - \sqrt{5}} = \frac{8\sqrt{2}\left(\sqrt{8} + \sqrt{5}\right)}{3} \quad = \frac{8}{3}\left(4 + \sqrt{10}\right)$

MATRIX–MATCH TYPE

1. A) $5^{2+4+6+\ldots\ldots 2x} = (25)^{28}$

$\Rightarrow 5^{x(x+1)} = 5^{56}$

$\Rightarrow x^2 + x - 56 = 0 \Rightarrow x = 7 \text{ as } x > 0$

B) $2\log_5 x = \log_{\sqrt{5}}\left(\frac{1/4}{1 - 1/2}\right)\log_5(0.2)$

$= \log_{\sqrt{5}}\left(\frac{1}{2}\right)\log_5\left(\frac{1}{5}\right)$

$= -\frac{\log_5\left(\frac{1}{2}\right)}{\log_5\sqrt{5}} = \log_5 4 \Rightarrow x = 2$

C) $\log x = \log_{2.5}\left(\frac{1/3}{1 - 1/3}\right)\log(0.16)$

$= \log_{5/2}(1/2)\log(2/5)^2$

$= \log 4$

$\Rightarrow x = 4$

D) $3^x \frac{(1/3)}{1 - 1/3} = \frac{2\left(5^2\right)}{1 - 1/5}$

$\Rightarrow \frac{1}{2}\left(3^x\right) = \frac{1}{2}\left(5^3\right)$

$\Rightarrow x = 3\log_3 5$

2. Let the roots be in A.P. and let they be a – b, a, a + b

then $(\alpha - \beta) + \alpha + (\alpha + \beta) = -b/a$

$\alpha(\alpha - \beta) + \alpha(\alpha + \beta) + \alpha^2 - \beta^2 = c/a$

$\alpha(\alpha - \beta)(\alpha + \beta) = d/a$

From first relation $\alpha = -\frac{b}{3a}$

From second relation $3\alpha^2 - \beta^2 = \frac{c}{a}$ and from third relation

$\alpha\left(\alpha^2 - \beta^2\right) = -\frac{d}{a}$

Since $\alpha = -\frac{b}{3a}$ we can easily eliminate β^2 in last two relation to get

$2b^3 - 9abc + 27a^2d = 0$

We can similarly arrive at

3. Since $b^2 = ac$

$2\log b = \log a + \log c$

$\Rightarrow$ log a, log b, log c are in A. P.

(A) $\log_p a, \log_p b, \log_p c$ in A.P.

(B) Since $\frac{1}{\log_p a}, \frac{1}{\log_p b}, \frac{1}{\log_p c}$ in H.P.

$\log_a p, \log_b p, \log_c p$ in H.P.

(C) since a, b, c in G.P.

and $\log_p c, \log_p b, \log_p a$ in A.P.

$a\log_p c, b\log_p c, c\log_p a$ in AGP

(D) on checking given number are in G.P.

4. Given $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{1}{2}$ -----(i)

$\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2} = \frac{9}{4}$ ......(ii)

$\alpha + \beta + \gamma = 2.$ -----(iii)

(A) from (i) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{1}{2}$

equering both sides

$$\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2} + 2\left(\frac{\alpha + \beta + \gamma}{\alpha\beta\gamma}\right) = \frac{1}{4}$$

or, $\frac{9}{4} + 2 \times \frac{2}{\alpha\beta\gamma} = \frac{1}{4}$ or, $\alpha\beta\gamma = -2$

(B) from equation (i) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{1}{2}$

or, $\frac{\beta\gamma + \alpha\gamma + \alpha\beta}{\alpha\beta\gamma} = \frac{1}{2}$ or, $\beta\gamma + \alpha\gamma + \alpha\beta = \frac{\alpha\beta\gamma}{2} = \frac{-2}{2} = -1$

(C) From (iii) $\alpha + \beta + \gamma = 2$

squaring both sides

$\alpha^2 + \beta^2 + \gamma^2 + 2(\alpha\beta + \beta\gamma + \alpha\gamma) = 4$
or, $\alpha^2 + \beta^2 + \gamma^2 + 2 \times -1 = 4$
or $\alpha^2 + \beta^2 + \gamma^2 = 6$
(D) as $\alpha^3 + \beta^3 + \gamma^3 - 3\alpha\beta\gamma = (\alpha + \beta + \gamma)[\alpha^2 + \beta^2 + \gamma^2 - (\alpha\beta + \beta\gamma + \gamma\alpha)]$
or, $\alpha^3 + \beta^3 + \gamma^3 - 3(-2) = 14$
or, $\alpha^3 + \beta^3 + \gamma^3 = 8$

ASSERTION – REASON TYPE

1. According to theory

$$t_r = \frac{1-x^r}{(1-x)^2} - \frac{rx^r}{1-x}$$

$$\sum_{r=1}^{n} t_r = \frac{1}{(1-x)^2}\sum_{r=1}^{n}(1-x^r) - \frac{1}{1-x}\sum_{r=1}^{n} rx^r$$

$$= \frac{1}{(1-x)^2}\left[n - \frac{x(1-x^n)}{1-x}\right] - \frac{1}{1-x}\left[\frac{x(1-x^n)}{(1-x)^2} - \frac{nx^{n+1}}{1-x}\right]$$

$$= \frac{n(1+x^{n+1})}{(1-x)^2} - \frac{2x(1-x^n)}{(1-x)^3}$$

2. $b = \frac{2ac}{a+c} \Rightarrow \frac{b}{a} = \frac{2c}{a+c}$ and $\frac{b}{c} = \frac{2a}{a+c}$

$$\frac{a+b}{2a-b} = \frac{1+b/a}{2-b/a} = \frac{a+3c}{2a}$$

and $\frac{c+b}{2c-b} = \frac{1+b/c}{2-b/c} = \frac{3a+c}{2c}$

Thus, $\frac{a+b}{2a-b} + \frac{c+b}{2c-b} \ge 1 + \frac{3}{2}\left(\frac{c}{a} + \frac{a}{c}\right) \ge 1 + \frac{3}{2}(2) = 4$

3. Rewrite the given expression as

$$\frac{1}{a} + \frac{1}{c-b} + \frac{1}{c} + \frac{1}{a-b} = 0$$

$$\Rightarrow \frac{a+c-b}{a(c-b)} + \frac{a+c-b}{c(a-b)} = 0$$

$$\Rightarrow a(c-b) = c(b-a) \Rightarrow b = \frac{2ac}{a+c}$$

For statement-2, put t = x/y, and write the expression as
$y^2 [a(b-c)t^2 + b(c-a)t + c(a-b)]$
t = 1 satisfies the expression within bracket. For perfect square other root must be 1.

4. Suppose $\sqrt{3}, \sqrt{5}$ and $\sqrt{7}$ are the pth, qth and rth terms of an A.P. whose common difference is d, then

$t_r - t_p = (r - p)\, d$

and $t_q - t_p = (q - p)\, d$

$\Rightarrow \frac{t_r - t_p}{t_q - t_p} = \frac{r-p}{q-p}$ which is rational numbers

$\Rightarrow \frac{\sqrt{7}-\sqrt{3}}{\sqrt{5}-\sqrt{3}}$ is a rational numbers.

$\Leftrightarrow \frac{\left(\sqrt{7}-\sqrt{3}\right)\left(\sqrt{5}+\sqrt{3}\right)}{5-3}$ is rational

$\Leftrightarrow \frac{\left(\sqrt{35}-\sqrt{15}\right)+\sqrt{21}-3}{2}$ is rational

$\Leftrightarrow \sqrt{35}-\sqrt{15}+\sqrt{21}$ is rational, say r.

Now,

$\sqrt{35}-\sqrt{15}+\sqrt{21} = r$

$\Rightarrow \sqrt{15}-\sqrt{21} = \sqrt{35} - r$

Squaring both sides, we get

$15+21-(2)(6)\sqrt{35} = 35 + r^2 - 2r\sqrt{35}$

$\Rightarrow \sqrt{35} = \frac{1-r^2}{12+2r}$

$\Rightarrow \sqrt{35}$ is rational

This is a contradiction

Hence $\sqrt{3}, \sqrt{5}$ and $\sqrt{7}$ cannot be three terms of an A.P.

5. a + c = 2b, ab = c^2 (i)

Now $\frac{2bc}{b+c} = \frac{c(a+c)}{b+c} = \frac{ac+ab}{b+c} = a$

$\therefore$ c, a,b are in H.P.

Eliminating a from two expressions in (i) we get

$c^2/b + c = 2b \Rightarrow c^2 + bc - 2b^2 = 0$

$(c-b)(c+2b) = 0 \Rightarrow c = -2b \ [\because c \neq b]$

Thus, a = 4b.

Now, a : b : c = 4 : 1 : –2

6. The Assertion A can be proved by taking the intersection of the inequaliaties.

a > 0, ar > 0, ar^2 > 0, a + ar > ar^2, ar + ar^2 > a.

ar^2 + a > ar

The inequalities follow from Reason

7. Statement -1

As A. M. $\geq$ G.M.

$$\frac{a}{b}+\frac{b}{c}\geq 2\sqrt{\frac{a}{b}\times\frac{b}{c}}$$

Or, $\frac{a}{b}+\frac{b}{c}\geq 2\sqrt{\frac{a}{c}}$ .....(i)

also $\frac{c}{d}+\frac{d}{e}\geq 2\sqrt{\frac{c}{e}}$ .....(ii)

Inequality (i) $\times$ (ii)

$$\left(\frac{a}{b}+\frac{b}{c}\right)\left(\frac{c}{d}+\frac{d}{e}\right)\geq 4\sqrt{\frac{a}{c}\times\frac{c}{e}}$$

or. $\left(\frac{a}{b}+\frac{b}{c}\right)\left(\frac{c}{d}+\frac{d}{e}\right)\geq 4\sqrt{\frac{a}{e}}$

statement (1) is correct

Statement (2)

As A. M. $\geq$ G.M.

$$\therefore \frac{b}{a}+\frac{c}{b}+\frac{d}{c}+\frac{e}{d}+\frac{a}{e}\geq 5\sqrt[5]{\frac{b}{a}\times\frac{c}{b}\times\frac{d}{c}\times\frac{e}{d}\times\frac{a}{e}}$$

or, $\frac{b}{a}+\frac{c}{b}+\frac{d}{c}+\frac{e}{d}+\frac{a}{e}\geq 5$

So, statement (2) is correct and is correct explanation for statement (1)

8. Statement (1)

as a, b, c, d, are in HP

'b' is the single H.M. between a and c

also A.M. between a and c is $\frac{a+c}{2}$

as, A.M. > H.M.

$$\therefore \frac{a+c}{2}>b$$

$\therefore$ a + c > 2b ....(i)

'c' is the single H.M. between b and d

A.M. between b and d is $\frac{b+d}{2}$

as , A.M. > H .M.

$$\frac{b+d}{2}>c$$

$\therefore$ b + d > 2c ....(ii)

inequality (i) + (ii)

a +c + b+ d > 2b +2c

a+ d > b +c so statment (1) is correct

Statement (2) as a, b, c, d, are in H.P. $\therefore \frac{1}{a},\frac{1}{b},\frac{1}{c},\frac{1}{d}$ will be in A.P.

$\therefore \frac{1}{b}-\frac{1}{a}=\frac{1}{d}-\frac{1}{c}$ or, $\therefore \frac{1}{a}+\frac{1}{d}=\frac{1}{b}+\frac{1}{c}$

So, statement (2) is correct and is not correct explanation for statement (1)

9. Let $t_r=\frac{r^2}{(2r-1)(2r+1)}$

$$\Rightarrow 4t_r=\frac{4r^2-1+1}{(2r-1)(2r+1)}$$

$$=1+\frac{1}{(2r-1)(2r+1)}$$

$$=1+\frac{1}{2}\left(\frac{1}{2r-1}-\frac{1}{2r+1}\right)$$

$$4\sum_{r=1}^{n}t_r=n+\frac{1}{2}\sum_{r=1}^{n}\left(\frac{1}{2r-1}-\frac{1}{2r+1}\right)$$

$$=n+\frac{1}{2}\left(1-\frac{1}{2n+1}\right) \qquad =n+\frac{n}{2n+1}=\frac{2n(n+1)}{2n+1}$$

$$\Rightarrow \sum_{r=1}^{n}t_r=\frac{n(n+1)}{2(2n+1)}$$

$\therefore$ Statement –1 is true

But statement-2 is false since

$$\Rightarrow \sum_{r=1}^{n}\frac{1}{(2r-1)(2r+1)}=\frac{1}{2}\sum_{r=1}^{n}\left(\frac{1}{2r-1}-\frac{1}{2r+1}\right)=\frac{1}{2}\left(1-\frac{1}{2n+1}\right)=\frac{n}{2n+1}$$

10. Suppose $\sqrt{3}, \sqrt{5}$ and $\sqrt{7}$ are the pth, qth and rth terms of an A.P. whose common difference is d, then

$t_r - t_p = (r - p)\, d$

and tq – tp = (q – p) d

$\Rightarrow \frac{t_r-t_p}{t_q-t_p}=\frac{r-p}{q-p}$ which is rational numbers

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Squaring both sides, we get

$15+21-(2)(6)\sqrt{35}=35+\mathrm{r}^2-2\mathrm{r}\sqrt{35}$

$\Rightarrow \sqrt{35}=\dfrac{1-\mathrm{r}^2}{12+2\mathrm{r}}$

$\Rightarrow \sqrt{35}$ is rational

This is a contradiction

Hence $\sqrt{3}, \sqrt{5}$ and $\sqrt{7}$ cannot be three terms of an A.P.

INTEGER ANSWER TYPE QUESTIONS

1. Let common ratio is $\dfrac{1}{2^b}$

and $S_\infty=\dfrac{a}{1-r}=\dfrac{\frac{1}{2^a}}{1-\frac{1}{2^b}}=\dfrac{1}{7}$

$\Rightarrow b=3 \,\&\, a=b$

$\Rightarrow b=3 \,\&\, a=b$

hence a = 3

2. for m = 91, n = (1111....91 times) is divisible by 3 since sum of digits is 91 which is ivisible by 3 hence n is not a prime number. Hence m - 87 = 4

3. Let $a=A-2d, x=A-d, y=A, z=A+d, b=A+2d$

givin $x+y+z=15 \Rightarrow A=5$

$\Rightarrow \mathrm{a}=5-2\mathrm{d}, \mathrm{b}=5+2\mathrm{d}$

also $\dfrac{1}{a}, \dfrac{1}{\alpha}, \dfrac{1}{\beta}, \dfrac{1}{\gamma}, \dfrac{1}{b}$ in A.P

$\Rightarrow \dfrac{1}{\alpha}+\dfrac{1}{\beta}+\dfrac{1}{\gamma}=\dfrac{3}{2}\left[\dfrac{1}{a}+\dfrac{1}{b}\right]=\dfrac{5}{3}$

or $\dfrac{3}{2}\left(\dfrac{1}{5-2\mathrm{d}}+\dfrac{1}{5+2\mathrm{d}}\right)=\dfrac{5}{3} \Rightarrow \mathrm{d}=\pm 2$

take d = -2 since a > b

hence a = 9

4. $$\sum_{K=1}^{n}\tan^{-1}\frac{2k}{1+\left(k^2+k+1\right)\left(k^2-k+1\right)}$$

$$\Rightarrow\sum_{K=1}^{n}\tan^{-1}\frac{\left(k^2+k+1\right)-\left(k^2-k+1\right)}{1+\left(k^2+k+1\right)\left(k^2-k+1\right)}$$

$$\Rightarrow\sum_{K=1}^{n}\tan^{-1}\left(k^2+k+1\right)-\tan^{-1}\left(k^2-k+1\right)$$

$$\tan^{-1}\left(n^2+n+1\right)-\tan^{-1}1$$

when $n\to\infty$

Then summation $\sum_{k=1}^{\infty}\tan^{-1}\frac{2k}{2+k^2+k^4}=\frac{\pi}{2}-\frac{\pi}{4}=\frac{\pi}{4}$

Hence $\frac{8}{\pi}\times\frac{\pi}{4}$ is 2

5. $$\frac{1}{\cos a_1\cos a_2}+\frac{1}{\cos a_2\cos a_3}+\ldots\ldots\frac{1}{\cos a_{n-1}\cos a_n}$$

$$\Rightarrow\frac{1}{\sin d}\left(\frac{\sin\left(a_2-a_1\right)}{\cos a_1\cos a_2}+\frac{\sin\left(a_3-a_2\right)}{\cos a_2\cos a_3}+\ldots\ldots\frac{\sin\left(a_n-a_{n-1}\right)}{\cos_{n-1}\cos_n}\right)$$

$$=\frac{1}{\sin \mathrm{d}}\left(\left(\tan \mathrm{a}_2-\tan \mathrm{a}_1\right)+\left(\tan \mathrm{a}_3-\tan \mathrm{a}_2\right)+\left(\tan \mathrm{a}_\mathrm{n}-\tan \mathrm{a}_{\mathrm{n}-1}\right)\right)$$

$$=\frac{1}{\sin d}\left(\tan a_n-\tan a_1\right)$$

$$\Rightarrow K=\frac{1}{\sin d}=2$$

6. here $\angle \mathrm{C}=90^0, \mathrm{A}+\mathrm{B}=90^0$

$$\mathrm{c}^2=\mathrm{a}^2+\mathrm{b}^2 \;\&\; 2\mathrm{b}=\mathrm{a}+\mathrm{c}$$

since $\mathrm{c}=2\mathrm{b}-\mathrm{a} \;\&\; \mathrm{c}^2=\mathrm{a}^2+\mathrm{b}^2$

$$\Rightarrow\left(2\mathrm{b}-\mathrm{a}\right)^2=\mathrm{a}^2+\mathrm{b}^2$$

or $\frac{\mathrm{b}}{\mathrm{a}}=\frac{4}{3}$

$\Rightarrow \dfrac{\sin B}{\sin A} = \dfrac{4}{3}$ or $\dfrac{\sin B + \sin A}{\sin B - \sin A} = \dfrac{7}{1}$

or $\cot \dfrac{B-A}{2} = \dfrac{1}{7} \Rightarrow \cos \dfrac{A-B}{2} = \dfrac{7}{5\sqrt{2}}$

Also 5(sinA+SinB) = $5\sqrt{2}\cos\dfrac{A-B}{2}$ =7

7. $T_2 = 3 + d, T_{10} = 3 + 9d, T_{34} = 3 + 33d$

since $T_2.T_{10}, T_{34}$ are in G.P

$T_{10}^{\ 2} = T_2 T_{34}$

$\Rightarrow (3 + 9d)^2 = (3 + d)(3 + 33d)$

$\Rightarrow d = 0, 1$

hence d = 1

8. According to question. total distance $= h + 2 \times \dfrac{2}{3} h + 2 \times \left(\dfrac{2}{3}\right)^2 h + 2 \times \left(\dfrac{2}{3}\right)^3 h + ...$ up to ∞

infimite

= $h + 2 \times \dfrac{2}{3} h(3)$

= $5h = 4500 cm$

$\Rightarrow 10h = 9000cm = 9 deca\ meters$

9. Let last term is T_n

$T_n = \dfrac{12}{23} + (n-1)\left(\dfrac{-14}{115}\right)$

= $\dfrac{74 - 14n}{115}$

hence for n = 5 last positive term obtained.

10. According to question,

$\dfrac{\log_z x}{\log_x y} = \dfrac{\log_y z}{\log_z x} \Rightarrow (\log x)^3 = (\log z)^3$

$\Rightarrow x = z$

Since $2y^3 = x^3 + z^3 \Rightarrow x^3 = y^3$ or $x = y$

given $xyz = 64 \,\&\, x = y = z$

$\therefore x = y = z = 4$

$\& x + y + z = 12$

11. $(1+4+7+10+..........up\,to\,25\,term)$

$+(2.3+5.6+8.9+........25terms)$

or $\sum_{x=1}^{25} 3r-2++\sum_{x=1}^{25} 3r(3r-1)$

= 49675

$\frac{S_{50}}{25} - 1980 = 7$

12. $\left(1+x+2x^2+3x^3+........+25x^{25}\right)\left(1+x+2x^2+3x^3+........+25x^{25}\right)$

$a_5 =$ coeff of $x^5 = 1.5+1.4+2.3+3.2+4.1+5$ = 30

$\Rightarrow \frac{a_5}{5} = 6$

13. Then x = a + (m -1) d and x = b r^{m-1}

y = a+(n-1)d and y = b r^{n-1}

z= a + (p-1) d and z = b r^{p-1}

$\therefore$ x - y = (m-n) d, y -z = (n-p) d, z -x = (p-m) d

Now $x^{y-z}y^{z-x}z^{x-y} = \left[br^{m-1}\right]^{(n-p)d}\left[br^{n-1}\right]^{(p-m)d}\left[br^{p-1}\right]^{(m-n)d}$

= $b^{[n-p+p-m+m-n]d}\ r^{[(m-1)(n-p)+(n-1)\ (p-m)+(m+n)]d}$

= $b^{0.d}$ $r^{0.d}$ = 1

14. Let the number of sides of the polygon be n Then the sum of all the interior angles = (n× 180 -360) sum of the interior angles .

= 120 + 125 + 130 .... to n terms

= $\frac{n}{2}\left[240+(n-1)5\right]$

$\therefore \frac{n}{2}\left[240+(n-1)5\right] = n \times 180^0 - 360^0$

$\Rightarrow$ n^2 - 25n + 144 = 0

$\Rightarrow$(n-9) (n-16) = 0

$\Rightarrow$n = 9 or 16

But when n = 16, the greatest interior angle is 120^0 +(16-1)5 =195^0 which is not possible, for interior angle is < 180.Hence the number of sides = 9.

15. $S_n = \sum_{k=1}^{n} k.2^{n+1-k} = 2^{n+1}\sum_{k=1}^{n} k.2^{-k}$

$$= 2^{n+1}\left[\frac{\frac{1}{2}\left(1-\frac{1}{2^n}\right)}{1-\frac{1}{2}}\right]\frac{1}{1-\frac{1}{2}}$$

$$= 2^{n+2}\left[1-\frac{1}{2^n}-\frac{n}{2^{n+1}}\right] = 2^{n+2} - 4 - 2n = 2\left(2^{n+1} - 2 - n\right)$$

so, $S_n = \frac{n+1}{4}(2^{n+1} - 2 - n)$ (as given)

So, $\frac{n+1}{4}\left(2^{n+1} - 2 - n\right) = 2\left(2^{n-1} - n - 2\right)$

n +1 = 8 , n =7

SUBJECTIVE QUESTIONS

1. Given that n = $p_1^{\alpha_1}.p_2^{\alpha_2}.p_3^{\alpha_3}.......p_k^{\alpha_k}$ ....(1)

where n $\in$ N and $p_1, p_2, p_3.....p_k$ are distinct prime numbers

Taking log on both sides of eq. (1), we get

log n =α_1 log p_1 + α_2 log p_2 + ...+ α_κ log p_k ....(2)

$p_1 \geq 2$

$\log_e$, pi $\geq \log_e 2$ ......(3)

$\forall i = 1, ...k$

Using (2) and (3) we get

log n $\geq$ α_1 log 2+ α_2 log 2+ α_3 log 2 +......+α_k log 2

$\Rightarrow$ log n $\geq$ (α_1 + α_2 +....+α_k) log 2

$\Rightarrow$ log n $\geq$ k log 2.

2. Suppose the given two numbers to be A and B. Then the n arithmetic means are $a_1, a_2, a_3, a_4, a_5,....................a_n$

So p = A + d = A + $\frac{B-A}{n+1}$

p = $\frac{nA+B}{n+1}$

For Harmonic progression, Suppose $x_1, x_2, ...x_n$ to be Harmonic means, then Harmonic progression for N +2 term is given by

A, $x_1, x_2, x_3,.....x_n$, B

Or q= I[st] Harmonic mean = $\frac{(n+1)AB}{nB+A}$

Now we have to prove that q does not lies between p and $\left(\frac{n+1}{n-1}\right)^2 p$

So the prove the given, we have to show that q is less than p. For this

Let $\frac{p}{q}=\frac{(nA+B)(nB+A)}{(n+1)^2 AB}$

Then $\frac{p}{q}-1=\frac{n\left(a^2+b^2\right)+ab\left(n^2+1\right)-(n+1)^2 ab}{(n+1)^2 ab}$

3. Let a be the first term and r be the common ration of the G.P. a_1, a_2, a_3,....... then $a_k = ar^{k-1}$ for k =1,2,3.... As a_1, a_2, ... are positive real numbers
a>0 , and r > 0
I case : when $r \neq 1$

we have $A_n = \frac{1}{n}(a_1 + a_2 + ...+a_n)$

$$\frac{1}{n}(a_1+a_2+......+ar^{n-1}) \qquad =\frac{1}{n}\left[\frac{a\left(1-r^n\right)}{1-r}\right]=\frac{a\left(1-r^n\right)}{n(1-r)}$$

$G_n = (a_1, a_2 ...a_n)^{\frac{1}{n}}$

and $\frac{1}{H_n}=\frac{1}{n}\left(\frac{1}{a_1}+\frac{1}{a_2}+...+\frac{1}{a_n}\right)=\frac{1}{n}\left(\frac{1}{a}+\frac{1}{ar}+...+\frac{1}{ar^{n-1}}\right) \Rightarrow H_n=\frac{n(1-r)ar^{n-1}}{1-r^n}$

Thus $A_n H_n = \frac{a(1-r^n)}{n(1-r)}.\frac{n(1-r)ar^{n-1}}{1-r^n} \qquad = a^2 r^{2(n-1)} = G_n^2$

Next, let G be geometric mean of G, G_2, ...G_n then
$G= (G_1 G_2 ...G_n)^{1/n}$
$\Rightarrow G^{2n}=(G_1G_2.....G_n)^2$

$= (A_1A_2.....A_n) (H_1H_2 .....H_n)=(A_1H_1)(A_2H_2)....(A_nH_n) \Rightarrow G=(A_1,A_2........A_nH_1H_2........H_n)^{1/2n}$
II case : When r = 1

In this case $A_n= \frac{1}{n}(a+a+....+a)=a$

Similarly G_n = a and H_n = a
Also $A_n H_n = a^2 = G_n^2$
In this case too,
$G= (A_1, A_2...A_n H_1 H_2 ....H_n)^{1/2n}$

4. A.M. $\geq$ G.M.

$$\frac{a+b+c+ab+bc+ca+abc}{7} \geq \left(a^4b^4c^4\right)^{\frac{1}{7}}$$

a +b+c +ab+bc+ca+abc $\geq$ 7$(a^4b^4c^4)^{1/7}$

$\Rightarrow$ 1 + a + b + c + ab + bc + ca + abc $\geq$ 7$(a^4b^4c^4)^{1/7}$

$\Rightarrow$(1+a) (1+b) (1+c)$\geq$ $^7(a^4b^4c^4)^{1/7}$

$\Rightarrow$)(1 +a)^7(1+b)^7(1+c)^7 $\geq 7^7(a^4b^4c^4)$

5. The first terms and common differences of q A.P. 's are 1,2,3,... q and 1,3,5,....... $(2q-1)$ (given)

$$S_1 = \frac{n}{2}\left[2.1+(n-1).1\right] = \frac{n}{2}\left[2+(n-1)\right]$$

$$S_2 = \frac{n}{2}\left[2.2+(n-1).3\right] = \frac{n}{2}\left[4+(n-1)3\right]$$

$$S_2 = \frac{n}{2}\left[2.3+(n-1)5\right] = \frac{n}{2}\left[6+(n-1)5\right]$$

....

$$S_q = \frac{n}{2}\left[2.q+(n-1)(2q-1)\right] = \frac{n}{2}\left[2q+(n-1)(2q-1)\right]$$

Adding all above equation, we get

$$S_1+S_2+S_3+.....+S_q = \frac{n}{2}\left[(2+4+6+...+2q)+(n-1)(1+3+5+...+(2q-1))\right]$$

$$= \frac{n}{2}\left[\frac{q}{2}(2+2q)+\frac{(n-1)q}{2}(1+2q-1)\right] \qquad = \frac{1}{2}nq(nq+1)$$

Alternative Method : If r is the first term of the A.P. then common difference is $(2r-1)$

$$\therefore \qquad S_r = \frac{n}{2}\left[2r+(n-1)(2r-1)\right] \quad = \frac{n}{2}\left[2nr-(n-1)\right]$$

$$S_r = n^2r - \frac{n(n-1)}{2}$$

$$\sum_{r=1}^{q} S_r = n^2\sum_{r=1}^{q} r - \frac{n(n-1)}{2}\sum_{r=1}^{q} 1 \quad = n^2\,\frac{q(q+1)}{2} - \frac{n(n-1)}{2}q$$

$$= \frac{nq}{2}\left[n(q+1)-(n-1)\right] \qquad = \frac{1}{2}nq(nq+1)$$

6. Let the three digits be a, ar and ar^2 then number

$100a+10ar+ar^2$ ....(1)

Given, $a + ar^2 = 2ar + 1$

$\Rightarrow$ $a(r-1)^2 = 1$

Also given, $a + ar = \frac{2}{3}\left(ar + ar^2\right)$

$\Rightarrow$ $2r^2 - r - 3 = 0$

$\therefore$ $r = -1, 3/2$

for $r = -1, a = \frac{1}{(r-1)^2} = \frac{1}{4} \notin I$ $\therefore r \neq -1$

for $r = 3/2, a = \frac{1}{\left(\frac{3}{2} - 1\right)^2} = 4$

From (1), number is $400 + 10.4.\frac{3}{2} + 4.\frac{9}{4} = 469$

7. Let the three numbers in G.P. be $\frac{a}{r}, a, ar$

then $\frac{a}{r} + a + ar = 70$ ..... (1)

and $\frac{4a}{r}, 5a, 4ar$ are is A.P.

$\therefore$ $10a = \frac{4a}{r} + 4ar$

$\Rightarrow$ $\frac{5a}{2} = 70 - a$ (from (1)]

$\therefore$ a= 20

from (1), $\frac{20}{r} + 20 + 20r = 70$

$\Rightarrow$ $2r^2 - 5r + 2 = 0$

$\therefore$ $r = 2$ or $\frac{1}{2}$

$\therefore$ The three numbers are 10,20,40 or 40,20,10

8. Let the common differences of three A.P.'s are d_1, d_2 *and* d_3

Given d_1, d_2, d_3 are in H.P.

$\therefore$ $\frac{1}{d_1}, \frac{1}{d_2}, \frac{1}{d_3}$ are is A.P.

$\therefore \quad \frac{1}{d_2}-\frac{1}{d_1}=\frac{1}{d_3}-\frac{1}{d_2}$ ..... (1)

Now $\quad s_1=\frac{n}{2}\{2.1+(n-1)d_1\}$

$\Rightarrow \quad s_1-n=\frac{(n-1)n}{2}d_1$

similarly $s_2-n=\frac{(n-1)nd_2}{2}$

and $\quad s_3-n=\frac{(n-1)nd_3}{2}$

substituting d_1, d_2 and d_3 in (1), then

$$\frac{n(n-1)}{2(s_2-n)}-\frac{n(n-1)}{2(s_1-n)}=\frac{n(n-1)}{2(s_3-n)}-\frac{n(n-1)}{2(s_2-n)}$$

$\Rightarrow \quad \frac{1}{s_2-n}-\frac{1}{s_1-n}=\frac{1}{s_3-n}-\frac{1}{s_2-n}$

$\Rightarrow \quad \frac{s_1-s_2}{(s_2-n)(s_1-n)}=\frac{s_2-s_3}{(s_2-n)(s_3-n)} \Rightarrow \frac{s_1-s_2}{s_1-n}=\frac{s_2-s_3}{s_3-n}$

$\Rightarrow \quad s_1s_3-ns_1-s_2s_3+ns_2=s_1s_2-s_1s_3-ns_2+ns_3$

$\Rightarrow \quad n(s_1-2s_2+s_3)=2s_3s_1-s_1s_2-s_2s_3$

$\therefore \quad n=\frac{2s_3s_1-s_1s_2-s_2s_3}{s_1-2s_2+s_3}$

9. p, q, r are in A.P.

$\therefore \quad q-p=r-q$

$\Rightarrow \quad p-q=q-r=k(let)$ ....(1)

given $\quad \frac{a-x}{px}=\frac{a-y}{qy}=\frac{a-z}{rz}$

$\Rightarrow \quad \frac{\frac{a}{x}-1}{p}=\frac{\frac{a}{y}-1}{q}=\frac{\frac{a}{z}-1}{r}$

$\Rightarrow \quad \dfrac{\left(\frac{a}{x}-1\right)-\left(\frac{a}{y}-1\right)}{p-q}=\dfrac{\left(\frac{a}{y}-1\right)-\left(\frac{a}{z}-1\right)}{q-r}$ (By law of Proportion)

$\Rightarrow \quad \dfrac{\frac{a}{x}-\frac{a}{y}}{k}=\dfrac{\frac{a}{y}-\frac{a}{z}}{k}$ (from (1))

$\Rightarrow \quad a\left(\frac{1}{x}-\frac{1}{y}\right)=a\left(\frac{1}{y}-\frac{1}{z}\right)$

$\therefore \quad \frac{2}{y}=\frac{1}{x}+\frac{1}{z}$

$\therefore \quad \frac{1}{x},\frac{1}{y},\frac{1}{z}$ are in A.P.

Hence x, y, z are in H.P.

10. From the first equation

$$\log_{10} x\left\{1+\frac{1}{2}+\frac{1}{4}+.......\infty\right\}=y$$

$\Rightarrow \quad \log_{10} x\left\{\frac{1}{1-1/2}\right\}=y$

$\Rightarrow \quad 2\log_{10} x=y$ .... (1)

from the second equation

$$\frac{1+3+5+....+(2y-1)}{4+7+10+...+(3y+1)}=\frac{20}{7\log_{10} x}$$

$\Rightarrow \quad \dfrac{\frac{y}{2}(1+2y-1)}{\frac{y}{2}(4+3y+1)}=\dfrac{20}{7\log_{10} x} \Rightarrow \dfrac{2y}{3y+5}=\dfrac{20}{7\log_{10} x}$

$\Rightarrow \quad 7y(2\log_{10} x)=60y+100$

$\Rightarrow \quad 7y(y)=60y+100$ $[from(1)]$

$\Rightarrow \quad y=10, y\neq\frac{-10}{7}$ (Since $y\in I^{+}$)

From (1), $2\log_{10} x = 10 \Rightarrow \log_{10} x = 5$

$\therefore \quad x = 10^5$

Hence required solution is

$x = 10^5, y = 10$

11. Let p and $(p+1)$ be removed number from 1,2,..., n then sum of remaining numbers

$$= \frac{n(n+1)}{2} - (2p+1)$$

From given condition $\frac{105}{4} = \frac{\frac{n(n+1)}{2} - (2p+1)}{n-2} \Rightarrow 2n^2 - 103n - 8p + 206 = 0$

Since n and p are integers so n must be even let n = 2r

we get $p = \frac{4r^2 + 103(1-r)}{4}$

Since p is an integer then $(1-r)$ must be divisible by 4. let $r = 1 + 4t,$ we get

$n = 2 + 8t$ and $p = 16t^2 - 95t + 1$, Now $1 \le p < n$

$\Rightarrow \quad 1 \le 16t^2 - 95t + 1 < 8t + 2 \Rightarrow$ t = 6

$\Rightarrow \quad n = 50$ and $p = 7$

Hence removed numbers are 7 and 8.

12. Let the nth term of the required progression be a_n its common difference being equal to d, then

$$S_x = \left(\frac{1st\ term + last\ term}{2}\right)x$$

$S_x = \frac{(a_1 + a_x)x}{2}$ ($\because$ first term a_1 and last term a_n)

$$S_{kx} = \frac{(a_1 + a_{kx})kx}{2}$$

Hence $\frac{S_{kx}}{S_x} = \left(\frac{a_1 + a_{kx}}{a_1 + a_x}\right)k$

$= \left(\frac{a_1 + a_1 + (kx-1)d}{a_1 + a_1 + (x-1)d}\right)k \quad = \left(\frac{2a_1 - d + kxd}{2a_1 - d + xd}\right)k$

For relation to be independent of x it is necesary & sufficient that

$2a_1 - d = 0 \Rightarrow d = 2a$

13 We have a, b,c are in A.P.

$\Rightarrow\ 2b = a + c$ .... (1)

α, β, γ are in H.P.

$\Rightarrow \quad \beta = \dfrac{2\alpha\gamma}{\alpha + \gamma}$ ....(2)

$a\alpha, b\beta, c\gamma$ are in G.P.

$\Rightarrow \quad b^2\beta^2 = a\alpha c\gamma$ .....(3)

Substituting the values of b and β from (1) and (2) , in (3) we get

$$\Rightarrow \quad \left(\frac{a+c}{2}\right)^2 \left(\frac{2\alpha\gamma}{\alpha+\gamma}\right)^2 = a\alpha c\gamma \Rightarrow \frac{(a+c)^2}{ac} = \frac{(\alpha+\gamma)^2}{\alpha\gamma}$$

$$\Rightarrow \quad \frac{a^2 + c^2 + 2ac}{ac} = \frac{\alpha^2 + \gamma^2 + 2\alpha\gamma}{\alpha\gamma} \Rightarrow \frac{a^2 + c^2}{ac} + 2 = \frac{\alpha^2 + \gamma^2}{\alpha\gamma} + 2$$

$$\Rightarrow \quad \frac{a^2 + c^2}{ac} = \frac{\alpha^2 + \gamma^2}{\alpha\gamma} \Rightarrow \alpha\gamma a^2 + \alpha\gamma c^2 = ac\alpha^2 + ac\gamma^2$$

$$\Rightarrow \quad a\alpha(a\gamma - c\alpha) - c\gamma(a\gamma - c\alpha) = 0 \Rightarrow (a\gamma - c\alpha)(a\alpha - c\gamma) = 0$$

$$\Rightarrow \quad aa - c\gamma \neq 0 \quad (\because a\alpha, c\gamma \text{ are distinct given})$$

$\therefore \quad a\gamma - c\alpha = 0$

using this in (3), $a\gamma = c\alpha$

$\Rightarrow \quad b^2\beta^2 = a^2\gamma^2$

from (4) and (5), $b\beta = a\gamma$

$$\frac{a}{(1/\gamma)} = \frac{b}{(1/\beta)} = \frac{c}{(1/\alpha)}$$

$$\therefore \quad a : b : c = \frac{1}{\gamma} : \frac{1}{\beta} : \frac{1}{\alpha}$$

14. Let G.P. be a, ar, ar^2,.. since the G.P. is infinite and decreasing $-1 < r < 1$ and $r > 0$, so $0 < r < 1$ and so a > 0. According to the hypthesis

$ar^2, 3a.ar^3, ar$ are in A.P. with common difference $\dfrac{1}{8}$.

$\Rightarrow\ 3a^2r^3 - ar^2 = \dfrac{1}{8}$ ..... (1) and $ar = ar^2 + 2.\frac{1}{8}$ .....(2)

$\Rightarrow$ $a = \frac{1}{4r(1-r)}$ .....(3)

from (1) and (3) $\frac{3r^3}{16r^2(1-r)^2} - \frac{r^2}{4r(1-r)} = \frac{1}{8}$

$\Rightarrow$ $\frac{3r^3}{16(1-r)^2} - \frac{r}{4(1-r)} = \frac{1}{8}$ $2r^2 + 3r - 2 = 0$

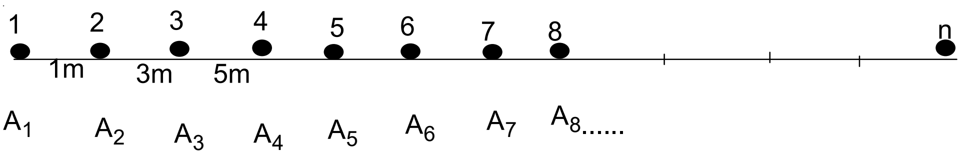
which gives $r = \frac{1}{2}, -2$ but $0 < r < 1$

$\therefore$ $r = \frac{1}{2}$ from (3), a= 1

Hence G.P. is $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, ...$

$\therefore$ $S_\infty = \frac{1}{1-\frac{1}{2}}$ =2

15. Distance covered by the person to bring first stone to the basket $D_1 = 0$



Distance covered by the person to bring second store to the basket

$D_2 = A_1A_2 + A_2A_1 = 1 + 1 = 2$ metre

Distance covered by the person to bring third stone to the basket

$D_3 = A_1A_3 + A_3A_1$ $= 2(1+3)$ metre

Distance covered by the person to bring fourth stone to the basket

$D_4 = A_1A_4 + A_4A_1$

$= 2(1+3+5)$ metre

....

....

Hence total distance covered by the person to bring n stones to the basket one by one

$D = D_1 + D_2 + D_3 + D_4 + ...n$ terms

$= 0 + 2 + 2(1+3) + 2(1+3+5) + ...(n-1)$ terms

$= 2\left[1 + (1+3) + (1+3+5) + ... + (n-1) terms\right]$

nth term of the series,

$$T_n = 2\left(1+3+5+....n\,terms\right)$$

$$= 2\frac{n}{2}\left(2.1+\left(n-1\right)2\right) \qquad T_n = 2n^2$$

$\therefore$ D = Sum of $\left(n-1\right)$ terms = $\sum_{n=1}^{n-1} 2n^2$

PREVIOUS IIT-JEE QUESTIONS

1. Using A.M. $\geq$ G.M.

$$\frac{(a+b)+(c+d)}{2} \geq \sqrt{(a+b)(c+d)} \Rightarrow \qquad 0 < M \leq 1$$

2. $\Rightarrow \qquad a_4 = a_1 + 3d = 2 + 3\times\frac{1}{9} = \frac{7}{3}$

$$\frac{1}{h_7} = \frac{1}{h_1} + 6d = \frac{7}{18}$$

3. $a_r = \frac{3}{4}$ and $\frac{a}{1-r} = 4$
4. Take sum of both series as given in question and make them equal
5. a,b, c, d are in A.P.

$\Rightarrow \qquad \frac{a}{abcd}, \frac{b}{abcd}, \frac{c}{abcd}, \frac{d}{abcd}$ are in A.P.

$\Rightarrow \qquad \frac{1}{bcd}, \frac{1}{acd}, \frac{1}{abd}, \frac{1}{abc}$ in A.P. $\Rightarrow$ bcd, acd, abd, abc in H.P.

6. x – 1 > 0 & x – 3 > 0 $\Rightarrow$ x > 3

$$\log_{2^2}(x-1) = \log_2(x-3) = \frac{1}{2}\log_2(x-1) = \log_2(x-3)$$

or $\qquad (x-1) = (x-3)^2$ or $x = 2, 5$

Hence solution is x = 5

7. 2b = a + c .....(i)

$b^4 = a^2c^2$ or $b^2 = \pm$ ac.....(ii)

here $\quad a+b+c = \frac{3}{2} \quad \Rightarrow \quad 3b = \frac{3}{2}$ or $b = \frac{1}{2}$ (from (i))

from (i), a + c =1 $\qquad ac = \pm b^2 = \pm\frac{1}{4}$ from (ii)

Solving this we get $a = \frac{1-\sqrt{2}}{2}$

8. $5 = \frac{x}{1-r} \Rightarrow r = \frac{5-x}{5}$

and $-1 < \frac{5-x}{5} < 1$ or $x \in (0,10)$

More than one correct choice

9. $x = \frac{1}{1-\cos^2\phi} = \frac{1}{\sin^2\phi}$ .....(i)

$y = \frac{1}{1-\sin^2\phi} = \frac{1}{\cos^2\phi}$ .....(ii)

$z = \frac{1}{1-\cos^2\phi\sin^2\phi}$ .....(iii)

$\Rightarrow \quad z = \frac{1}{1-\frac{1}{y}.\frac{1}{x}} = \frac{xy}{xy-1}$ or $xyz = xy + z$

also xy = x + y

10. Find a and d from the given data and get answer

11. x, y z are in G.P.

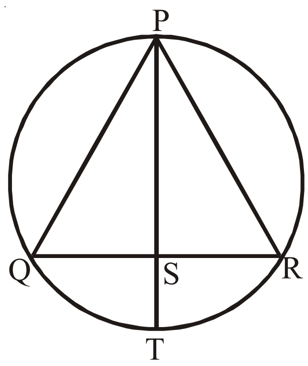
$\Rightarrow$ lnx, lny, lnz are in A.P.

$\Rightarrow$ 1 + lnx, 1 + lny, 1 + lnz are in A.P.

$\Rightarrow \quad \frac{1}{1+\ln x}, \frac{1}{1+\ln y}, \frac{1}{1+\ln z}$ are in H.P.

12. By generaly, PS × = 2S × SR .....(i)

(B) since for unequal real positive number G.M. > H.M.



$\sqrt{PS\times ST} =< \frac{2}{\frac{1}{PS}+\frac{1}{PT}}$

$\Rightarrow \quad \frac{1}{PS}+\frac{1}{PT} > \frac{2}{\sqrt{PS\times ST}} = \frac{2}{\sqrt{QS\times SR}}$

(D) Also $\sqrt{QS\times SR} < \frac{QS+2R}{2}$ $(\because G.M. < A.M.)$

$\Rightarrow \quad \frac{2}{QS+SR} < \frac{1}{\sqrt{QS\times SR}} < \frac{1}{2}\left(\frac{1}{PS}+\frac{1}{ST}\right)$

$\Rightarrow \quad \frac{1}{PS}+\frac{1}{PT} > \frac{4}{QR}$

13. for n = 1, $S_1 = \frac{1}{3} = 0.3$ & $T_1 = \frac{1}{1} = 1$

Also $\frac{\pi}{3\sqrt{3}} = \frac{\pi\sqrt{3}}{9} = \frac{3.14\times 1.7o3}{9} = 0.58$

$\Rightarrow \quad S_1 < \frac{\pi}{3\sqrt{3}} < T_1 \Rightarrow S_n < \frac{\pi}{3\sqrt{3}}$ & $T_n > \frac{\pi}{3\sqrt{3}}$

14. $V_r = \frac{r}{2}[2\times r + (r-1)(2r-1)] = \frac{r}{2}[2r^2 - r + 1] = r^3 - \frac{r^2}{2} + \frac{r}{2}$

$$\sum_{r=1}^{n} V_r = \sum_{r=1}^{n} r^3 - \frac{r^2}{2} + \frac{r}{2} = \frac{n(n+1)(3n^2+n+2)}{12}$$

15. $T_r = V_{r+1} - V_{r-2}$
= (3r – 1) (r + 1) = composite number.

16. $Q_r = T_{r+1} - T_r$
= 6r + 5
and $d = Q_r - Q_{r-1} = 6$

17. $A_1 = \frac{a+b}{2},\ a_1 = \sqrt{ab},\ H_1 = \frac{2ab}{a+b}$

$A_n = \frac{A_{n-1}+H_{n-1}}{2}, G_n = \sqrt{A_{n-1}H_{n-1}} = H_n = \frac{2A_{n-1}H_{n-1}}{A_{n-1}+H_{n-1}}$

since $G_n^2 = A_nH_n = A_{n-1}H_{n-1}$

$\Rightarrow \quad A_nH_n = A_{n-1}H_{n-1} = A_{n-2}H_{n-2} = A_1H_1$

$\Rightarrow \quad G_n^2 = G_{n-1}^2 = G_{n-2}^2 \quad = G_1^2 = ab$

18. $A_n - A_{n-1} = \frac{A_{n-1}+H_{n-1}}{2} - A_{n-1} = \frac{H_{n-1}-A_{n-1}}{2} < 0$

$\Rightarrow \quad A_n < A_{n-1}$ or $A_1 > A_2 > A_3 \ldots$

19. Since $G_n^2 = A_nH_n = ab$

$H_n = \frac{ab}{A_n}$

since $A_1 > A_2 > A_3 > \frac{ab}{H_1} > \frac{ab}{H_2} > \frac{ab}{H_3} \ldots. \Rightarrow \quad H_1 < H_2 < H_3$

20. Let $a_1 = a$, $a_2 = a_5$, $a_3 = ar^2$, $a_4 = ar^3$
$b_1 = a$, $b_2 = a + ar$, $b_3 = a + ar + ar^2$, $b_4 = a + ar + ar^2 + ar^3$. So b_1, b_2, b_3, b_4 are neither in A.P. nor in G.P. nor in H.P. So, statement-1 is true and statement 2 is false.

21. Since n A.M.'s have been inserted between a and b C.d of A.P. $d = \frac{b-a}{n+1}$

Now p = first A.M. = 2nd term of A.P. $= a + d = a + \frac{b-a}{n+1} = \frac{an+b}{n+1}$ …..(i)

Since n H.M.'s has been inserted between a and b C.d. of corresponding A.P.

$$d_1 = \frac{\frac{1}{b} - \frac{1}{a}}{n+1} = \frac{a-b}{ab(n+1)}$$

Now q = first H.M. = 2nd term of H.P.

$\therefore \quad \frac{1}{q}$ = 2nd term of corresponding A.P. = $\frac{1}{a} + d = \frac{1}{a} + \frac{a-b}{ab(n+1)} = \frac{bn+a}{ab(n+1)}$

$\therefore \quad q = \frac{ab(n+1)}{bn+a}$ ….(ii)

We have to show that q cannot lie between p and $[(n + 1)/(n - 1)]^2$ P.

Since $n+1 > n-1, \left(\frac{n+1}{n-1}\right)^2 P > P$

Now $\frac{P}{q} = \frac{n^2ab + ab + nb^2 + na^2}{(n+1)^2 ab}$

$\Rightarrow \quad \frac{P}{q} - 1 = \frac{n(a^2 + b^2 - 2ab)}{(n+1)^2 ab} = \frac{n}{(n+1)^2}\left(\sqrt{\frac{b}{a}} - \sqrt{\frac{a}{b}}\right)^2 \Rightarrow \frac{P}{q} - 1 > a$

as $P < [(n + 1)/(n - 1)]^2P$, it follows that q cannot lies between P and $[(n + 1)]^2P$.

22. S_n = Sum of an infinite G.P. whose first term is n and common ratio $r = \frac{1}{n+1}$

$S_n = \frac{a}{1-r} = \frac{n}{1-\left(\frac{1}{n+1}\right)} = n+1$. Putting n = 1, 2, 3, …..2n – 1

$S_1 = 2, S_2 = 3$ ….. $S_{2n-1} = 2n$

$S_1^2 = 2^2, S_2^2 = 3^2 .....S_{2n-1}^2 = (2n)^2$

$\therefore \quad S_1^2 + S_3^2 + ... + S_{2n-1}^2$

$= 2^2 + 3^2 + 4^2 + ... + (2n)^2$

$= 1^2 + 2^2 + 3^2 + 4^2 + ... + (2n)^2 - 1^2 = \frac{2n(2n+1)(4n+1)}{6} - 1$

23. Since x_1, x_2, x_3 are in A.P.

Therefore x_1 = a- d, x_2 = a and x_3 = a + d and x_1, x_2, x_3 are the roots of $x^3 - x^2 + \beta x + \gamma = 0$

We have $\Sigma\alpha = 1$ ... (i)

$\Sigma\alpha\beta = \beta$ ... (ii)

$\Sigma\alpha\beta\gamma = -\gamma$ ... (iii)

from (i) a = 1/3

from (ii) $3a^2 - d^2 = \beta$

$\Rightarrow 1/3 - \beta = d^2 \Rightarrow 1/3 - \beta \geq 0 \; (\because d^2 \geq 0) \Rightarrow \beta \in (-\infty, 1/3]$

from (iii), a $(a^2 - d^2) = -\gamma$ $\Rightarrow$ $\frac{1}{3}\left(\frac{1}{9}-d^2\right)=-\gamma$

$\Rightarrow \gamma+\frac{1}{27}\geq 0 \Rightarrow \gamma\in\left[-\frac{1}{27},\infty\right)$

24. Any four consecutive integers in A.P. be taken as $(a-3d), (a-d), (a+d), (a+3d)$. so that c.d. is 2d and their product is $(a^2-9d^2)(a^2-d^2)$.

$= (2d)^4 + (a^2 - 9d^2)(a^2 - d^2)$

Since a and d are given to be integers therefore $(a^2 - 5d^2)^2$ is also an integer.

25. $a_1, a_2 \ldots\ldots a_n$ are in G.P.

$\therefore \quad a_2 = a_1r,\ a_3 = a_1r^2,\ a_4 = a_1r^3 \ldots\ldots a_n = a_1r^{n-1}$ .....(i)

$$A_n=\frac{\Sigma a_1}{n}=\frac{a_1}{n}\left[1+r+r^2+....+r^{n-1}\right]=\frac{a_1}{n}\left(\frac{r^n-1}{r-1}\right) \quad .....(ii)$$

$$G_n=\left(a_1a_2.....a_n\right)^{\frac{1}{n}}=\left[a_1^n\left(1.r.r^2.....r^{n-1}\right)\right]^{\frac{1}{n}}=a_1r^{\frac{n-1}{2}} \quad .....(iii)$$

$$\frac{1}{1+n}=\frac{1}{n}\Sigma\frac{1}{a_1} \qquad \therefore \quad H_n=\frac{na_1r^{n-1}(r-1)}{r^{n-1}} \quad .....(iv)$$

from (ii), (iii) & (iv) we get $\quad G_n^2=A_nH_n=a_1^2r^{n-1}$

Above is true for each n,

Hence $\quad G_1^2G_2^2.....G_n^2=(A_1A_2.....A_n)(H_1H_2.....H_n)$

$$\therefore \quad \left(G_1G_2.....G_n\right)^{\frac{1}{n}}=\left[\left(A_1A_2.....A_n\right)\left(H_1H_2.....H_n\right)\right]^{\frac{1}{2n}}$$

26. a, A_1, A_2, b are in A.P. $\qquad A_1 + A_2 = a + b$

A, G_1, G_2, b are in G.P. $\qquad G_1.G_2 = ab$

A, H_1, H_2, b are in H.P.

$$H_1=\frac{3ab}{2b+a},\ H_2=\frac{3ab}{2ab} \qquad \Rightarrow \qquad \frac{1}{H_1}+\frac{1}{H_2}=\frac{1}{a}+\frac{1}{b}$$

$$\frac{H_1+H_2}{H_1H_2}=\frac{A_1+A_2}{G_1G_2}=\frac{1}{a}+\frac{1}{b} \quad .....(i)$$

Now $$\frac{G_1G_2}{H_1H_2}=\frac{ab}{\left(\frac{3ab}{2b+a}\right)\left(\frac{3ab}{2a+b}\right)}=\frac{(2a+b)(a+2b)}{9ab} \quad .....(ii)$$

27. $2b = a + c$ are in A.P. $\Rightarrow$ $4b^2 = a^2 + c^2 + 2ac$

$b^2(a^2 + c^2) = 2a^2c^2$ $\Rightarrow$ $b^2(a^2 + c^2) = 2a^2c^2$ are in H.P.

$b^2(4b^2 - 2ac) = 2a^2c^2$ $\Rightarrow$ $\left(b^2+\frac{ac}{2}\right)\left(b^2-ac\right)=0$

$$b^2 + \frac{ac}{2} = 0 \quad b^2 - ac = 0$$

28. $$\frac{a+b+c+ab+bc+ca+abc}{7} \ge \left(a^4b^4c^4\right)^{\frac{1}{7}}$$

$\Rightarrow$ $1 + a + b + c + ab + bc + ca + abc > 7(a^4b^4c^4)^{1/7}$

29. $$-A_n = -\frac{3}{4}\left(\frac{3}{4}\right)^2 - \left(\frac{3}{4}\right)^3 + ... + (-1)^n\left(\frac{3}{4}\right)^n$$

30. $$S_k = \frac{\frac{k-1}{k!}}{1-\frac{1}{k}} = \frac{1}{(k-1)!}$$

We have $S_1 = 1$

$S_2 = 1$

$S_3 = \frac{1}{2}$

Now, $\sum_{k=1}^{100}\left|\left(k^2 - 3k + 1\right)S_k\right|$

$= S_1 + S_2 + S_3 + \sum_{k=4}^{100}\frac{\left(k^2-3k+1\right)}{\left(k-1\right)!} = 1 + 1 + \frac{1}{2} + \sum_{k=4}^{100}\left[\frac{1}{\left(k-3\right)!} - \frac{1}{\left(k-1\right)!}\right]$

$= 1 + 1 + \frac{1}{2} + \left[1 + \frac{1}{2!} - \frac{1}{98!} - \frac{1}{99!}\right] = 4 - \frac{100}{99!}$

So, $\frac{100^2}{100!} + \sum_{k=1}^{100}\left|\left(k^2-3k+1\right)S_k\right| = 4$.

31. $a_1^2 + a_2^2 + a_3^2 + ... + a_{11}^2 = 990$

$\Rightarrow a^2 + (a + d)^2 + (a + 2d)^2 + ... + (a + 10d)^2 = 990$

$\Rightarrow 11a^2 + d^2 (1^2 + 2^2 + 3^2 + ... + 10^2) \quad + ad (2 + 4 + 6 + ... + 20) = 990$

$\Rightarrow 11 \times 225 + d^2 \times 385 + d \times 15 \times 110 = 990 \ \Rightarrow 7d^2 + 30d + 27 = 0$

$\Rightarrow d = -3, -9/7$ (n.p.)

$\therefore$ $d = -3$ and $a_1 = 15$

$\therefore \ \frac{a_1 + a_2 + ... + a_{11}}{11} = \frac{11}{2\times 11}(2\times 15 + 10\times(-3)) = 0$