HINTS & SOLUTIONS

EXERCISE - 1

NEET LEVEL

1. (A) Slope $\frac{dy}{dx} = 3x^2 - 6x - 9$

if tangent is parallel to the x–axis then $\frac{dy}{dx} = 0$

$\Rightarrow 3x^2 - 6x - 9 = 0 \quad \Rightarrow x^2 - 2x - 3 = 0$

$\Rightarrow x^2 - 3x + +x - 3 = 0 \quad \Rightarrow (x-3)(x+1) = 0$

$\Rightarrow x = 3 \text{ or } x = -1 \quad \Rightarrow y = -20 \text{ or } y = 12$

2. (A) Enclosed area : $A = \pi r^2$

So $\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

Here $r = 8$ cm, $\frac{dr}{dt} = 5$ cm/s

$\Rightarrow \frac{dA}{dt} = (2\pi)(8)(5) = 80\pi \text{ cm}^2/\text{s}$

3. (B) $\because p = t\ell nt$

$\therefore F = \frac{dp}{dt} = \frac{d}{dt}(t\ell nt) = (1)\ell nt + (t)\left(\frac{1}{t}\right) = 1 + \ell nt$

$F = 0 \Rightarrow 1 + \ell nt = 0 \Rightarrow \ell nt = -1 \Rightarrow t = e^{-1} = \frac{1}{e}$

4. (C) Check $\vec{A}.\vec{B} = 0$

5. (A) Let side of cube be x then $\frac{dx}{dt} = 3$ cm/s

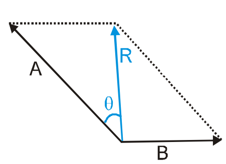
$\because V = x^3 \therefore \frac{dV}{dt} = 3x^2 \frac{dx}{dt} = 3 \times 10^2 \times 3 = 900 \text{ cm}^3/\text{s}$

6. (B) Resultant $= \sqrt{3^2 + 4^2 + 12^2} = \sqrt{5^2 + 12^2} = 13N$

7. (B) $\sqrt{(0.5)^2 + (-0.8)^2 + c^2} = 1$

$\Rightarrow 0.25 + 0.64 + c^2 = 1 \Rightarrow c^2 = 0.11 \Rightarrow c = \pm\sqrt{0.11}$

8. (A) Let forces be A and B and B < A then A + B = 16



$A\cos\theta = R = 8$ and $A\sin\theta = B$

$\Rightarrow A^2 = 8^2 + B^2 \quad \Rightarrow A^2 - B^2 = 64$

$\Rightarrow (A-B)(A+B) = 64 \quad \Rightarrow A - B = 4$

$\Rightarrow A = 10N \;\&\; B = 6N$

9. (A) Required unit vector

$= \frac{\vec{A}+\vec{B}}{|\vec{A}+\vec{B}|} = \frac{3\tilde{i} + 6\tilde{j} - 2\tilde{k}}{\sqrt{3^2 + 6^2 + 2^2}} = \frac{1}{7}(3\tilde{i} + 6\tilde{j} - 2\tilde{k})$

10. (B) **11. (B)**

12. (D) For zero resultant, sum of any two forces ≥ remaining force

13. (A) $\vec{a} = \vec{c} + \overrightarrow{RP}$ and $\vec{b} = \vec{c} + \overrightarrow{RQ}$ but $\overrightarrow{RP} = -\overrightarrow{RQ}$

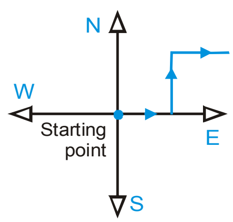
$\Rightarrow \vec{a} + \vec{b} = 2\vec{c} + \overrightarrow{RP} + \overrightarrow{RQ} \Rightarrow \vec{a} + \vec{b} = 2\vec{c}$

14. (B) $\vec{R} = \vec{P} + \vec{Q},\ \vec{R}' = \vec{P} + 2\vec{Q}$

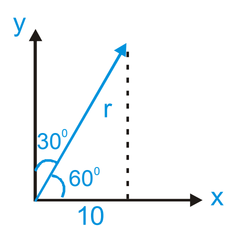
$\because \vec{R}'\cdot\vec{P} = 0 \therefore (\vec{P} + 2\vec{Q})\cdot\vec{P} = 0 \Rightarrow P^2 + 2\vec{Q}.\vec{P} = 0$

$R^2 = P^2 + Q^2 + 2\vec{P}.\vec{Q} = P^2 + Q^2 - P^2 = Q^2 \Rightarrow R = Q$

15. (A)



16. (D) $\cos 60^0 = \frac{10}{r} \Rightarrow r = \frac{10}{1/2} = 20$ units



17. (D) $\tilde{v} = \frac{(4-1)\tilde{i} + (2+2)\tilde{j} + (3-3)\tilde{k}}{\sqrt{(4-1)^2 + (2+2)^2 + (3-3)^2}} = \frac{3}{5}\tilde{i} + \frac{4}{5}\tilde{j}$

$\tilde{v} = (10)\left(\frac{3}{5}\tilde{i} + \frac{4}{5}\tilde{j}\right) = 6\tilde{i} + 8\tilde{j}$

18. (A) Use $R^2 = A^2 + B^2 + 2AB\cos\theta$ or see options

19. (C)

20. (B) Required angle $= \frac{2\pi}{12} = \frac{360}{12} = 30^0$

21. (C) Displacement $= \sqrt{12^2 + 5^2 + 6^2}$

$= \sqrt{144 + 25 + 36} = \sqrt{205} = 14.31$ m

22. (D) $\because \quad |\vec{A} \times \vec{B}| = \sqrt{3}\ \vec{A} \cdot \vec{B} \quad \therefore$

$AB\sin\theta = \sqrt{3}AB\cos\theta$

$\Rightarrow \quad \tan\theta = \sqrt{3} \quad \Rightarrow \quad \theta = 60°$

$|\vec{A} + \vec{B}| = \sqrt{A^2 + B^2 + 2AB\cos 60°}$

$= \sqrt{A^2 + B^2 + 2AB\left(\frac{1}{2}\right)} = \sqrt{A^2 + B^2 + AB}$

23. (B)

24. (B) $\because \quad \vec{P} + \vec{Q} = \vec{R} \qquad \therefore \ \vec{Q} = \vec{R} - \vec{P}$

$\Rightarrow \quad Q^2 = R^2 + P^2 - 2RP\cos\theta_1$

$\Rightarrow \quad \cos\theta_1 = \frac{1}{2} \ \Rightarrow \ \theta_1 = \frac{\pi}{3}$

Now $\quad \because \ \vec{P} + \vec{Q} + \vec{R} = \vec{0} \quad \therefore \vec{P} + \vec{R} = -\vec{Q}$

$\Rightarrow \quad P^2 + R^2 + 2PR\cos\theta_2 = Q^2$

$\Rightarrow \quad \cos\theta_2 = -\frac{1}{2} \ \Rightarrow \ \theta_2 = \frac{2\pi}{3}$

25. (B) $\because \quad \vec{A}.\vec{B} = AB\cos\theta$

$\therefore$ Projection of $\vec{A}$ on $\vec{B} = A\cos\theta = \frac{\vec{A}.\vec{B}}{B} = \vec{A}.\hat{B}$

26. (A) Resultant $= \sqrt{(x^2 + y^2)}$

$= \sqrt{(x+y)^2 + (x-y)^2 + 2(x+y)(x-y)\cos\theta}$

$\Rightarrow \quad x^2 + y^2 = 2(x^2+y^2) + 2(x^2-y^2)\cos\theta$

$\Rightarrow \quad \cos\theta = \frac{1}{2}\left(\frac{x^2 + y^2}{y^2 - x^2}\right)$

27. (D) Projection on x–y plane $= \sqrt{3^2 + 1^2} = \sqrt{10}$

28. (A) **29. (D)**

30. (A) Velocity of one ball $\vec{v}_1 = \hat{i} + \sqrt{3}\hat{j}$

Velocity of second ball $\vec{v}_2 = 2\hat{i} + 2\hat{j}$

Angle between their path :

$$\cos\theta = \frac{\vec{v}_1.\vec{v}_2}{v_1 v_2} = \frac{2 + 2\sqrt{3}}{(2)(2\sqrt{2})} = \frac{1 + \sqrt{3}}{2\sqrt{2}} \Rightarrow \theta = 15°$$

31. (A) In a clockwise system $\hat{k} \times \hat{j} = \hat{i}$

32. (B) $|\vec{e}_1 - \vec{e}_2| = \sqrt{1^2 + 1^2 - 2(1)(1)\cos\theta} = 2\sin\frac{\theta}{2}$

34. (A) $\vec{v} = \vec{\omega} \times \vec{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 2 \\ 0 & 4 & -3 \end{vmatrix}$

$= \hat{i}\ (6-8) - \hat{j}\ (-3) + \hat{k}\ (4) = -2\hat{i} + 3\hat{j} + 4\hat{k}$

$|\vec{v}| = \sqrt{2^2 + 3^2 + 4^2} = \sqrt{4 + 9 + 16} = \sqrt{29}$

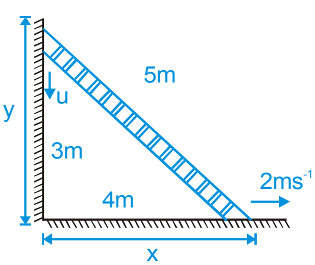
EXERCISE - 2

AIIMS LEVEL

1. (A) $x^2 + 4 = y \Rightarrow 2xdx = dy$ but $dy = 2dx$

So $2xdx = 2dx \Rightarrow 2x = 2 \Rightarrow x = 1 \Rightarrow y = 1^2 + 4 = 5$

2. (C) At any instant $x^2 + y^2 = 5^2$



Differentiating w.r.t. time $2x\frac{2x}{dt} + 2y\frac{dy}{dt} = 0$

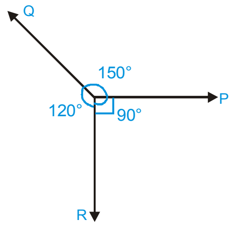
Here $\frac{dx}{dt} = 2, \ \frac{dy}{dt} = u \ \Rightarrow \ u = \frac{8}{3}$ m/s

3. (C) $I = \frac{2}{5}MR^2 = \frac{2}{5}\left(\frac{4}{3}\pi R^3 \rho\right)R^2 = \frac{8}{15}\pi\rho R^5$

$\frac{dI}{dt} = \left(\frac{8}{15}\pi\rho\right)(5R^4)\frac{dR}{dt} = \left(\frac{8\pi}{15}\right)\left(\frac{M}{4/3\ \pi R^3}\right)(5R^4)$

$\frac{dR}{dt} = 2MR\left(\frac{dR}{dt}\right) = (2)(1)(1)(2) = 4$ kg m^2s^{-1}

4. **(D)**



$$\frac{P}{\sin 120°} = \frac{Q}{\sin 90°} = \frac{R}{\sin 150°}$$

$$\Rightarrow \frac{P}{\sqrt{3}/2} = \frac{Q}{1} = \frac{R}{1/2}$$

$$\Rightarrow \frac{2P}{\sqrt{3}} = \frac{Q}{1} = \frac{2R}{1} = k \text{ (constant)}$$

$$\Rightarrow P:Q:R = \frac{\sqrt{3}k}{2} : k : \frac{k}{2} = \sqrt{3}:2:1$$

5. **(B)** $\because |\vec{a}+\vec{b}| = 1 \quad \therefore 2\cos\frac{\theta}{2} = 1$

$$\Rightarrow \cos\frac{\theta}{2} = \frac{1}{2} \Rightarrow \frac{\theta}{2} = \frac{\pi}{3}$$

$$|\vec{a}-\vec{b}| = 2\sin\frac{\theta}{2} = 2\times\frac{\sqrt{3}}{2} = \sqrt{3}$$

6. **(D)** $\because |\hat{a}+\hat{b}+\hat{c}| = 1$

$\therefore |\hat{a}|^2 + |\hat{b}|^2 + |\hat{c}|^2 + 2(\hat{a}\cdot\hat{b} + \hat{b}\cdot\hat{c} + \hat{c}\cdot\hat{a}) = 1$

$\Rightarrow 1 + 1 + 1 + 2(\cos\theta_1 + \cos\theta_2 + \cos\theta_3) = 1$

$\Rightarrow \cos\theta_1 + \cos\theta_2 + \cos\theta_3 = -1$

7. **(D)** $|\vec{a}+\vec{b}+\vec{c}| = \sqrt{a^2 + b^2 + c^2 + 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a})}$

$\because \vec{a}\cdot(\vec{b}+\vec{c}) = 0,\ \vec{b}\cdot(\vec{c}+\vec{a}) = 0\ \&\ \vec{c}\cdot(\vec{a}+\vec{b}) = 0$

$\Rightarrow \vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a} = 0$

$\Rightarrow |\vec{a}+\vec{b}+\vec{c}| = \sqrt{a^2+b^2+c^2} = \sqrt{3^2+4^2+5^2} = 5\sqrt{2}$

8 **(C)** $a_x = 2a_y$, $\cos\gamma = \frac{a_z}{a} = \cos 135° = -\frac{1}{\sqrt{2}}$

$$\Rightarrow a_z = -\frac{a}{\sqrt{2}} = -\frac{5\sqrt{2}}{\sqrt{2}} = -5$$

Now $a_x^2 + a_y^2 + a_z^2 = 50 \Rightarrow 4a_y^2 + a_y^2 + 25 = 50$

$\Rightarrow a_y^2 = 5 \Rightarrow a_y = \pm\sqrt{5} \Rightarrow a_x = \pm 2\sqrt{5}$

9. **(B)**

10. **(B)** $\because \vec{C} = \vec{A} + \vec{B} \quad \therefore C^2 = A^2 + B^2 + 2AB\cos\theta$

If $C^2 < A^2 + B^2$ then $\cos\theta < 0$.

Therefore $\theta > 90^0$

11. **(B)** $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4$

$= (4\hat{i} - 5\hat{j} + 5\hat{k}) + (-5\hat{i} + 8\hat{j} + 6\hat{k}) + (-3\hat{i} + 4\hat{j} - 7\hat{k})$

$+ (12\hat{i} - 3\hat{j} - 2\hat{k}) = 4\hat{j} + 2\hat{k}$

$\Rightarrow$ motion will be in y–z plane

12. **(B)** Area of triangle $= \frac{1}{2}(\vec{a}\times\vec{b}) = \frac{1}{2}(\vec{b}\times\vec{c}) = \frac{1}{2}(\vec{c}\times\vec{a})$

13. **(C)** $\vec{\tau} = \vec{r}\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7 & 3 & 1 \\ -3 & 1 & 5 \end{vmatrix} = 14\hat{i} - 38\hat{j} + 16\hat{k}$

14. **(A)** Here $\alpha = 45°$ so inclination of AC with x–axis is 45°.
So unit vector along AC

$$= \cos 45°\hat{i} + \sin 45°\hat{j} = \frac{\hat{i}+\hat{j}}{\sqrt{2}}$$

15. **(A)** Displacement $d\vec{r} = dx\hat{i} + dy\hat{j}$

but $3y + kx = 5$ so $3dy + kdx = 0$

$$\Rightarrow d\vec{r} = dx\hat{i} - \frac{k}{3}dx\hat{j} = \left(\hat{i} - \frac{k}{3}\hat{j}\right)dx$$

Work done is zero if $\vec{F}.d\vec{r} = 0$

$$(2\hat{i}+3\hat{j})\cdot\left(\hat{i} - \frac{k}{3}\hat{j}\right)dx = 0 \Rightarrow (2-k)dx = 0 \Rightarrow k=2$$

16. **(A)** For triangle ABC : $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{0}$

Now $\quad \overrightarrow{AB} + \overrightarrow{BC} + 2\overrightarrow{CA}$

$= \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} + \overrightarrow{CA} = \vec{0} + \overrightarrow{CA} = \overrightarrow{CA}$

17. **(C)** $(\vec{a} + 3\vec{b}).(7\vec{a} - 5\vec{b}) = 0$

$\Rightarrow 7a^2 - 15b^2 + 16\,\vec{a}\cdot\vec{b} = 0 \qquad$ **...(i)**

and $\quad (\vec{a} - 4\vec{b})\cdot(7\vec{a} - 2\vec{b}) = 0$

$\Rightarrow 7a^2 + 8b^2 - 30\,\vec{a}\cdot\vec{b} = 0 \qquad$ **...(ii)**

By adding (i) and (ii)

$\Rightarrow -23b^2 + 46\,\vec{a}\cdot\vec{b} = 0 \Rightarrow 2\vec{a}\cdot\vec{b} = b^2$

So $7a^2 - 15b^2 + 8b^2 = 0 \Rightarrow a^2 = b^2$

$\Rightarrow 2ab\cos\theta = b^2 \Rightarrow 2\cos\theta = 1$

$\Rightarrow \theta = \cos^{-1}(1/2) = 60°$

EXERCISE - 3

P-1 (Matrix Match)

1. (A) – q, (B)–r, (C)–s, (D)–s
2. (A) $\to$ r, (B) $\to$ p, (C) $\to$ q, (D) $\to$ s
3. (A) $\to$ q, (B) $\to$ r, (C) $\to$ p, (D) $\to$ s

EXERCISE - 3

P-2 (Assertion & Reason)

1. A **2.** C **3.** C **4.** D **5.** B **6.** A
7. C **8.** B **9.** D **10.** A **11.** A **12.** A

EXERCISE - 4

P-1 (NEET/AIPMT)

1. A **2.** D **3.** D **4.** D

EXERCISE - 4

P-2 (AIIMS)

1. A

MOCK TEST

1. (C) Since x = 0 is one of the solution so the product will be zero.

2. (B) $\log(-2x) = 2\log(x+1)$

$-2x > 0 \Rightarrow x < 0$ .....(i)

$x + 1 > 0 \Rightarrow x > -1$ .....(ii)

from (i) & (ii), we get $x \in (-1, 0)$

$\therefore\ -2x = (x+1)^2 \Rightarrow x^2 + 4x + 1 = 0$

$$\Rightarrow x = \frac{-4 \pm 2\sqrt{3}}{2} = -2 \pm \sqrt{3}$$

so $x = -2 + \sqrt{3}$ only one solution lies in (–1, 0)

3. (C) $\log_2 15\ \log_{1/6} 2\ \log_3 1/6 =$

$$\frac{\log 15}{\log 2} \times \frac{\log 2}{\log 1/6} \times \frac{\log 1/6}{\log 3}$$

$$= \frac{\log(3\times 5)}{\log 3} = 1 + \log_3 5 > 2 \text{ (but } < 3)$$

4. (B) Case I

$[x] - 2x = 4$ ... (i)

$\Rightarrow [x] - 2([x] + \{x\}) = 4$

$\Rightarrow [x] + 2\{x\} + 4 = 0$ ... (ii)

$\because\ 0 \le 2\{x\} < 2$

$\therefore\ 0 \le -[x] - 4 < 2 \Rightarrow -6 < [x] \le -4$

$\Rightarrow [x] = -4, -5$

$\therefore$ from (i) we get $x = -4, \dfrac{-9}{2}$

Case II

$[x] - 2x = -4$ ...(iii)

$\Rightarrow [x] = 2x - 4$

$\Rightarrow [x] = 2([x] + \{x\}) - 4$

$\Rightarrow 2\{x\} = 4 - [x]$ ... (iv)

$\therefore\ 0 \le 2\{x\} < 2$

$\Rightarrow 0 \le 4 - [x] < 2$

$\Rightarrow 2 < [x] \le 4 \quad \therefore [x] = 3, 4$

$\therefore$ from (iii) we get $x = 4, \dfrac{7}{2}$

$\therefore$ Number of solutions of $|[x] - 2x| = 4$ are 4.

5. (C) (i) $\log_{\frac{1}{3}}(x^2 + x + 1) > -1 \Rightarrow x^2 + x + 1 < 3$

$\Rightarrow x^2 + x - 2 < 0 \Rightarrow (x+2)(x-1) < 0$

$\Rightarrow x \in (-2, 1)$ ....(1)

and **(ii)** $x^2 + x + 1 > 0 \Rightarrow x \in R$ ....(2)

by (1) & (2) $x \in (-2, 1)$

6. (A) $5\{x\} = x + [x]$ .... (i)

$[x] - \{x\} = \dfrac{1}{2}$ .... (ii)

$\because\ 0 \le \{x\} < 1$

$\Rightarrow 0 \le [x] - \dfrac{1}{2} < 1$ (by (ii)

$\Rightarrow [x] = 1 \quad \therefore \{x\} = \dfrac{1}{2}$

$\therefore$ from (i) we get $\dfrac{5}{2} = x + 1$

$\therefore\ x = \dfrac{3}{2}$, (one value)

7. (D) $|x^2 - 9| + |x^2 - 4| = 5$

$|x^2 - 9| + |x^2 - 4| = |(x^2 - 9) - (x^2 - 4)|$

$\Rightarrow (x^2 - 9)(x^2 - 4) \le 0\ \{\because |a| + |b| = |a - b| \Leftrightarrow a.b \le 0\}$

$\Rightarrow x \in [-3, -2] \cup [2, 3]$

8. (B) Here $x \ne 0$

Case I when $x \ge -2$

$\frac{|x+2|-x}{x} < 2 \quad \Rightarrow \frac{2}{x} < 2$

$\Rightarrow \frac{1}{x} < 1 \Rightarrow (x-1)/x > 0$

$x \in [-2, 0) \cup (1, \infty)$ ...(i)

Case II when $x < -2$

$\frac{|x+2|-x}{x} < 2 \quad \Rightarrow \frac{-2-2x}{x} < 2 \quad \Rightarrow \frac{1+x}{x} + 1 > 0$

$\Rightarrow (1+2x)/x > 0 \Rightarrow x \in (-\infty, -2)$ ...(ii)

$\therefore$ from (i) and (ii) we get $x \in (-\infty, 0) \cup (1, \infty)$

9. **(D)** $0 \le \log_e[2x] \le 1$

$1 \le [2x] \le e \Rightarrow [2x] = 1, 2 \quad \Rightarrow 1 \le 2x < 3$

$\therefore \frac{1}{2} \le x < \frac{3}{2}$

10. **(A)** $|a| + |b| = |a - b| \quad \Rightarrow a.b \le 0$

$(x^2 - 5x + 7)(x^2 - 5x - 14) \le 0$

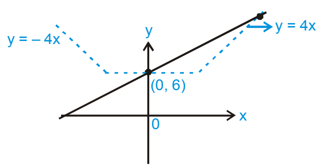
$(x-7)(x+2) \le 0 \quad \Rightarrow x \in [-2, 7]$

11. **(B)** When **(i)** $P = 0$ then it has infinte slution

(ii) if $-4 < P < 0$ or $0 < P < 4$

then it intersect at 2 points

(iii) $P \ge 4$ or $P \le -4$ then it has only one solution



12. **(C)** use $A^{\log_A B} = B$

Basic (E)

$e^{\ell n(\ell n 3)} = \ell n 3$

$\therefore \quad e^{e^{\ell n(\ell n 3)}} = e^{\ell n 3} = 3$

13. **(D)** $0 = (\vec{a} + \vec{b}) \cdot (2\vec{a} + 3\vec{b}) \times (3\vec{a} - 2\vec{b})$

$= (\vec{a} + \vec{b}) \cdot (-4\vec{a} \times \vec{b} - 9\vec{a} \times \vec{b}) = -13\,(\vec{a} + \vec{b}) \cdot (\vec{a} \times \vec{b})$

which is true for all values of $\vec{a}$ and $\vec{b}$.

14. **(D)** Volume of the parallelopiped formed by $\vec{a}', \vec{b}', \vec{c}'$ is 4

$\therefore$ Volume of the parallelopiped formed by $\vec{a}, \vec{b}, \vec{c}$ is $\frac{1}{4}$

$\vec{b} \times \vec{c} = \frac{(\vec{c}' \times \vec{a}') \times \vec{c}}{4} = \frac{1}{4}\vec{a}'$

$\therefore \quad |\vec{b} \times \vec{c}| = \frac{\sqrt{2}}{4} = \frac{1}{2\sqrt{2}}$

$\therefore$ length of altitude $= \frac{1}{4} \times 2\sqrt{2} = \frac{1}{\sqrt{2}}$.

15. **(B)** Let $\vec{a} = \lambda\vec{b} + \mu\vec{c}$

then $\frac{\vec{a}.\vec{b}}{ab} = \frac{\vec{a}.\vec{d}}{ad}$

i.e. $\frac{(\lambda\vec{b} + \mu\vec{c}).\vec{b}}{b} = \frac{(\lambda\vec{b} + \mu\vec{c}).\vec{d}}{d}$

i.e. $\frac{[\lambda(2\hat{i} + \hat{j}) + \mu(\hat{i} - \hat{j} + \hat{k})].(2\hat{i} + \hat{j})}{\sqrt{5}}$

$= \frac{[\lambda(2\hat{i} + \hat{j}) + \mu(\hat{i} - \hat{j} + \hat{k})].(\hat{j} + 2\hat{k})}{\sqrt{5}}$

i.e. $\lambda(4+1) + \mu(2-1) = \lambda(1) + \mu(-1+2)$ i.e. $4\lambda = 0$

i.e. $\lambda = 0$

$\therefore \quad \hat{a} = \frac{\hat{i} - \hat{j} + \hat{k}}{\sqrt{3}}$

16. **(C)** $\vec{r} \cdot \vec{a} = 0$, $|\vec{r} \times \vec{b}| = |\vec{r}|\ |\vec{b}|$ and $|\vec{r} \times \vec{c}| = |\vec{r}||\vec{c}| \Rightarrow \vec{r} \perp \vec{a}, \vec{b}, \vec{c}$

$\therefore \vec{a}, \vec{b}, \vec{c}$ are coplaner. $\therefore [\vec{a}\ \vec{b}\ \vec{c}] = 0$

17. **(C)** **Since** $\vec{a}_1$, $\vec{a}_2$ & $\vec{a}_3$ are non-coplanar vectors

$\therefore \quad x + y - 3 = 0$ ... (i)

$2x - y + 2 = 0$ ... (ii)

$2x + y + \lambda = 0$ ... (iii)

From (i) & (ii) $x = 1/3$, $y = 8/3$

$\therefore$ from (iii) $\lambda = -\frac{10}{3}$

18. **(D)** $\vec{a}' = \frac{\vec{b} \times \vec{c}}{[\vec{a}\vec{b}\vec{c}]} = \frac{\hat{i} + \hat{j} - \hat{k}}{2}$

19. **(A)** $\vec{a} + 2\vec{b} + 3\vec{c} = \vec{0} \quad \Rightarrow \vec{a} \times \vec{b} + 3\vec{c} \times \vec{b} = \vec{0}$

i.e. $\vec{a} \times \vec{b} = 3\vec{b} \times \vec{c}$, $\vec{a} \times \vec{c} + 2\vec{b} \times \vec{c} = \vec{0}$

i.e. $2\vec{b}\times\vec{c} = \vec{c}\times\vec{a}$

$\therefore\ \vec{a}\times\vec{b} + \vec{b}\times\vec{c} + \vec{c}\times\vec{a} = 3\vec{b}\times\vec{c} + \vec{b}\times\vec{c}$

$+2\vec{b}\times\vec{c} = 6\vec{b}\times\vec{c}$

20. (B) Let $\vec{r} = \ell(\vec{b}\times\vec{c}) + m(\vec{c}\times\vec{a}) + n(\vec{a}\times\vec{b})$

$\vec{r}\,.\,\vec{a} = \ell\,[\vec{a}\ \vec{b}\ \vec{c}] \Rightarrow \ell = 1$

similarly $m = 2$, $n = 3$

$\therefore\ \vec{r} = (\vec{b}\times\vec{c}) + 2(\vec{c}\times\vec{a}) + 3(\vec{a}\times\vec{b})$

21. (C) Since $\vec{a}, \vec{c}, \vec{b}$ form a right-handed system, therefore

$$\vec{c} = \vec{b}\times\vec{a} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 1 & 0 \\ x & y & z \end{vmatrix} = z\vec{i} - x\vec{k}$$

22. (A) S1 : Obvious

S2 : $(4\hat{i}+7\hat{j}-2\hat{k}) - (3\hat{i}-4\hat{j}+7\hat{k}) = \hat{i}+11\hat{j}-9\hat{k}$

$\therefore$ form a triangle .

S3 : $\vec{a}\ .\ (\vec{a}+\vec{b})\times(\vec{a}+\vec{b}+\vec{c}) = \vec{a}\,.\,(\vec{a}+\vec{b})\times\vec{c}$

$= [\vec{a}\ \vec{b}\ \vec{c}]$

S4 : $(\vec{a}\times\vec{b})\times\vec{c} = \vec{a}\times(\vec{b}\times\vec{c}) \Rightarrow (\vec{b}\,.\,\vec{c})\,\vec{a}$

$= (\vec{a}\,.\,\vec{b})\,\vec{c} \Rightarrow (\vec{b}\,.\,\vec{c})\,\vec{a} - (\vec{a}\,.\,\vec{b})\,\vec{c} = \vec{0}$

$\vec{c}\times(\vec{a}\times\vec{b}) = (\vec{b}\,.\,\vec{c})\,\vec{a} - (\vec{a}\,.\,\vec{c})\,\vec{b}$

$= (\vec{a}\,.\,\vec{b})\,\vec{c} - (\vec{a}\,.\,\vec{c})\,\vec{b} = (\vec{b}\times\vec{c})\times\vec{a} \neq \vec{0}$

23. (B) S1 : $\vec{a}$ and $\lambda\vec{a}$ are parallel vectors.

S2 : $\vec{a}\,.\,\vec{b}$ may take negative values also.

S3 : $|(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})| = |-(\vec{a}\times\vec{b})+(\vec{b}\times\vec{a})|$

$= 2\,|\vec{b}\times\vec{a}|$

S4 : $(\vec{a}\times\vec{b})^2 = (\vec{a}\times\vec{b})\ .\ (\vec{a}\times\vec{b})$

$= \vec{a}\ .\ (\vec{b}\times(\vec{a}\times\vec{b})) = \vec{a}\ .\ \left((\vec{b}.\vec{b})\,\vec{a} - (\vec{a}.\vec{b})\,\vec{b}\right)$

$= (\vec{a}\,.\,\vec{a})\,(\vec{b}\,.\,\vec{b}) - (\vec{a}\,.\,\vec{b})(\vec{b}\,.\,\vec{a})$

24. (D) S1 : $|\vec{a}| = |\vec{b}| = |\vec{a}-\vec{b}| = 1$

$a^2 + b^2 - 2\vec{a}\,.\,\vec{b} = 1$

$\therefore$ angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{3}$

S2 : $\frac{\vec{a}+\vec{b}}{2}$ A vector in the direction of angle bisector of $\vec{a}$ and $\vec{b}$ is $\vec{a}+\vec{b}$

$\therefore$ the given statement is not correct in general

S3 : $(\vec{a}\,.\,\hat{i})^2 + (\vec{a}\,.\,\hat{j})^2 + (\vec{a}\,.\,\hat{k})^2 = |\vec{a}|^2 = \vec{a}^2$

S4 : Any vector in the plane $\hat{i}+\hat{j}+\hat{k}$ and $-\hat{i}+\hat{j}+\hat{k}$ is of the form

$\alpha(\hat{i}+\hat{j}+\hat{k}) + \beta(-\hat{i}+\hat{j}+\hat{k}) = (\alpha-\beta)\hat{i} + (\alpha+\beta)\hat{j} + (\alpha+\beta)\hat{k}$

this will be perpendicular with $\hat{i}-\hat{j}-\hat{k}$

if $(\alpha-\beta)-(\alpha+\beta)-(\alpha+\beta) = 0 \Rightarrow \alpha = -3\beta$

Hence the required vector is of the form $(-4\hat{i} - 2\hat{j} - 2\hat{k})$

$\therefore$ statement is false

25. A → (q), B → (p), C → (t), D → (r)

By using definitions of modulus, greatest integer and fractional part function, obviously.

26. A → (t), B → (p), C → (q), D → (s)

(A) $\vec{a}+\vec{b} = \hat{j}$ and $2\vec{a}-\vec{b} = 3\hat{i}+\frac{\hat{j}}{2}$

$\therefore\ \vec{a} = \hat{i}+\frac{\hat{j}}{2},\ \vec{b} = -\hat{i}+\frac{\hat{j}}{2}\quad \therefore \cos\theta = -\frac{3}{5}$

(B) $|\vec{a}+\vec{b}+\vec{c}| = \sqrt{6}$

$\Rightarrow\ \vec{a}^2+\vec{b}^2+\vec{c}^2+2\vec{a}\cdot\vec{b}+2\vec{b}\cdot\vec{c}+2\vec{c}\cdot\vec{a} = 6$

$\therefore\ |\vec{a}| = 1$

(C) $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -2 \\ 1 & -3 & 4 \end{vmatrix} = -2\hat{i}-14\hat{j}-10\hat{k}$

$\therefore$ Area $= 5\sqrt{3}$

(D) $\vec{a}$ is perpendicular

$\vec{b}+\vec{c} \Rightarrow \vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}=0$ .... (i)

$\vec{b}$ is perpendicular $\vec{a}+\vec{c} \Rightarrow \vec{b}\cdot\vec{c}+\vec{b}\cdot\vec{c}=0$ .... (ii)

$\vec{c}$ is perpendicular $\vec{a}+\vec{b} \Rightarrow \vec{c}\cdot\vec{a}+\vec{c}\cdot\vec{b}=0$ .... (iii)

from (i), (ii) and (iii) we get

$\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{c}=\vec{c}\cdot\vec{a}=0$

$\therefore\ |\vec{a}+\vec{b}+\vec{c}|=7$

27. **(D)** Statement 1 is false

$\because$ Sum of the length of any two sides of a triangle is greater than length of third side

Statement 2 is true

$\because\ a^2+c^2-b^2<0$

then cos B < 0 $\Rightarrow$ B is obtuse

28. **(C)** The result can be easily understood with the help of nature of graph of $y=\log_a x$

29. **(B)** Both the statements are correct but statement–2 is not correct explanation of statement–1 because

vectors $\vec{b}, \vec{c}, \vec{d}$ in statement–1 are coplanar.

30. **(D)** Statement–1 is false and Statement–2 is true.

Since $\vec{a}\,.(\vec{b}\times\vec{c})=0$

$\therefore\ \vec{a}, \vec{b}, \vec{c}$ are coplanar